Multi-node Offer Stack Optimisation over Electricity Networks

A. Downward, D. Tsai, Y. Weng and G. Zakeri
Department of Engineering Science
University of Auckland, New Zealand
a.downward@auckland.ac.nz

Abstract

In this work we examine the problem that electricity generators face when offering power at multiple locations into an electricity market. The amount of power offered at each node can affect the price at the other node, so it is important to optimise both offers simultaneously.

Even with perfect information (i.e. known demand, and known offers from competitors) this is a non-convex bi-level optimisation problem. We first show how this can be formulated as an integer program using special ordered sets of type 2 (SOS2) enabling this problem to be solved efficiently.

We then extend this work to allow for uncertainty, and hence find the profit maximising offer stacks at each node (as opposed to a single quantity, as in the deterministic case).

We demonstrate the intuition that we can gain from this model in a simple two-node example, and conclude with future work.
Key words: electricity markets, integer programming, bi-level optimization.

1 New Zealand electricity market

In the New Zealand electricity market, every half-hour generators submit offers, in the form of offer stacks (a collection of up to five tranches, each specifying a quantity of power and a corresponding price). These offers are used by the system operator (Transpower) to determine how much electricity each generator should produce (dispatch) as well as nodal power prices at every location in the country.

These dispatch quantities are determined through a large linear program, which has the objective of minimising the cost of generation, while meeting the demand over the country, while complying with network constraints. This model is known as SPD, which stands for scheduling, pricing and dispatch.

In this paper, we will present an optimisation model which strategic generators can use to determine what offer stacks to submit to the system operator in order to maximise their profits. Initially, we will present this model in the context of a single node, and show how an optimal offer stack can be constructed. We will then extend this model over networks, and present a two-node example to gain some intuition into the strategic incentives that networks provide. We conclude by discussing some extensions to this model.

1.1 Scheduling, pricing and dispatch

We will initially introduce the SPD model for a single-node. This will allow us to develop the method for constructing offers stacks clearly, before we extend the model to handle networks, where generators potentially simultaneously offer at multiple nodes, in section 4.

Sets and indices

$t \in \mathcal{T}$: the set of all offered tranches.

Parameters

q_t = offer quantity for tranche t;
p_t = offer price for tranche t;
d = market demand.

Decision variables

x_t = the dispatched quantity for tranche t;
π = the market price.

Single-node Dispatch Model

$$
\begin{aligned}
\min \quad & \sum_{t \in \mathcal{T}} p_t x_t \\
\text{s.t.} \quad & \sum_{t \in \mathcal{T}} x_t = d \qquad [\pi] & (1) \\
& 0 \le x_t \le q_t \quad \forall t \in \mathcal{T}. & (2)
\end{aligned}
$$

Explanation

The objective is to minimise the total cost of dispatched tranches. Constraint (1) requires plants to produce enough power to satisfy the demand; the shadow price of this constraint is π, the clearing price. Constraints (2) ensures that each dispatch quantity is within the bounds of the corresponding tranche.

2 Single-node optimisation

In this section we will consider the behaviour of a profit-maximising generator at a single node.

2.1 Generator offer model

A strategic generator, wishing to maximise its profits would seek to solve a bi-level optimization problem. This problem would have the optimal dispatch prices and quantities (as a function of the strategic generator's offer) embedded within the set of constraints. Essentially a generator would solve:

$$
\begin{aligned}
\max \quad & y \times \pi \\
\text{s.t.} \quad & 0 \leq y && (3) \\
& y \leq C && (4) \\
& \min \quad \sum_{t \in \mathcal{T}} p_t x_t \\
& \text{s.t.} \quad \sum_{t \in \mathcal{T}} x_t = d - y \qquad [\pi] \\
& \qquad\quad 0 \leq x_t \leq q_t \quad \forall t \in \mathcal{T}.
\end{aligned}
$$

Constraints (3) and (4) give the bounds on the strategic generation quantity y. The embedded linear program gives the correct value of π (the clearing price) as a function of the generation y.

In its current form this problem would be extremely difficult to solve, firstly, since the objective is not linear, it is bilinear; and secondly we have an optimisation problem within the constraints.

One option for solving this could be to find the optimality (KKT) conditions of the dispatch problem, and embed those within the above optimisation problem as either complementarity constraints, or using binary variables and big-M constraints. The latter method was implemented by Nates (2010), along with an approximation of the objective function and was found to be rather inefficient. Philpott et al. (2005) have also considered this problem using dynamic programming, albeit for a generator situated at a single node.

2.2 Piecewise-linear reformulation

Here we will present an alternate approach which is both efficient and requires no approximation. Consider the dispatch problem presented earlier. From the optimality conditions of this problem it can be seen that a solution to this problem can be found by simply ordering the tranches from the cheapest to the most expensive. Moreover, if there exists any partially dispatched tranches the market price must equal that tranche's offer price, otherwise the price must be greater than all fully-dispatched tranches and less than all undispatched tranches. These conditions on price can be shown to be equivalent to simply choosing a point on an offer stack sorted from cheapest to most expensive. Let us consider the simple offer stack shown in figure 1; this offer stack depicts 4 tranches.

We can parametrise the offer stack, as a function of t, the distance travelled from the origin. The piecewise-linear functions $p(t)$ and $q(t)$ are shown in figure 2. Using these piecewise-linear functions, we can rewrite the optimisation problem for a generator as follows:

$$
\begin{aligned}
\max \quad & y \times \pi \\
\text{s.t.} \quad & q(t) = d - y \\
& p(t) = \pi \\
& 0 \leq y \leq C.
\end{aligned}
$$

Note that we have not shown the full formulation of the piecewise-linear functions. These can be modelled using *special ordered sets of type 2*, or by utilising the

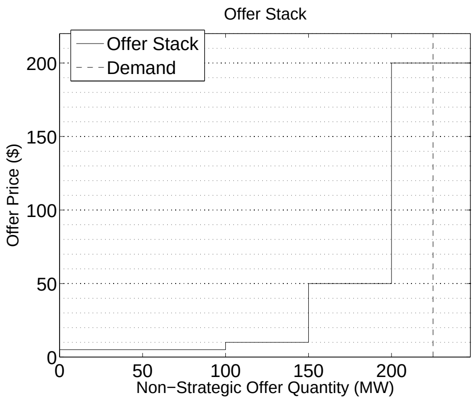

Figure 1: Example of a typical offer stack.

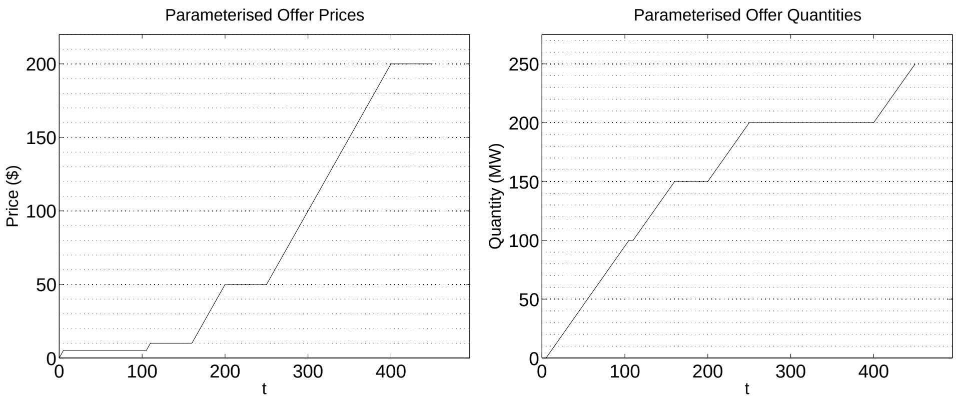

Figure 2: Price and quantity parameterised in t.

piecewise-linear functionality of a modelling language such as AMPL (with CPLEX). Note, however, that this model still has a bilinear term in the objective. This can be made linear, as follows:

$$\begin{aligned} y\pi &= (d - q(t))\, p(t) \\ &= dp(t) - q(t)\, p(t), \end{aligned}$$

where $d - q(t)$ is the residual demand at price $p(t)$, or the demand remaining after all other generators offering less than $p(t)$ are dispatched.

On the surface, this still appears nonlinear, since we have the term $q(t)\, p(t)$. However, recall that $(q(t), p(t))$ is the parametric representation of a piecewise-constant function. This means that whenever $q(t)$ is increasing, $p(t)$ is constant, and vice-versa. Thus we can define: $pq(t) := q(t) \times p(t)$, and $pq(t)$ will be a piecewise-linear function of t. This is shown in the left graph of figure 3; on the right is a plot of the strategic generator's revenue $(dp(t) - pq(t))$ as a function of t. This leaves us with the following formulation, which can be solved using a standard integer programming solver.

$$\begin{aligned} \max \quad & dp(t) - pq(t) \\ \text{s.t.} \quad & q(t) = d - y \\ & 0 \le y \le C. \end{aligned}$$

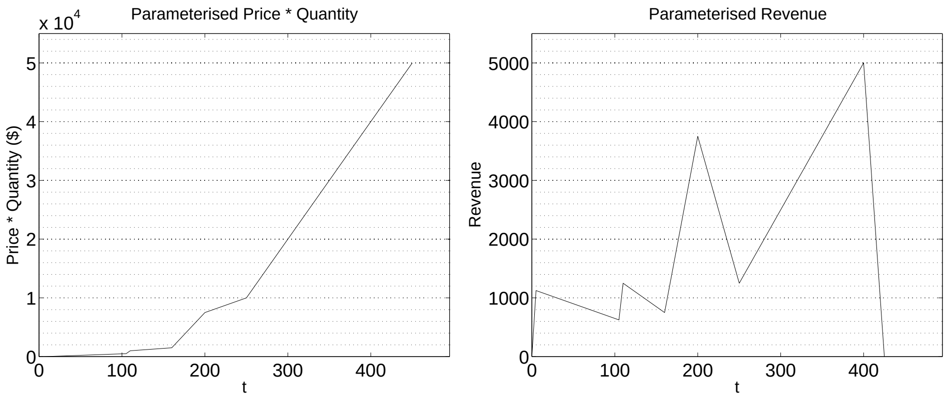

Figure 3: Revenue of the parametric formulation.

3 Creating an optimal offer stack

So far, we have only considered optimising for a single scenario. This results in a single optimal price and quantity which maximises profit. In fact, when you submit your offer, you are not sure what offers have been submitted by other firms, and there will be uncertainty with respect to wind generation, as well as changes in demand. This means that instead of simply choosing a single quantity that maximizes profit, you want to submit an offer stack that maximises expected profit. We will encapsulate this uncertainty, for each scenario $\omega \in \Omega$ by using a superscript, e.g. the demand in scenario ω would be d^ω.

3.1 Monotonicity

In order for the offer stack that you submit to the market to be valid, it must be monotonically increasing. This requirement means that it is not possible to construct an offer that will be optimal for each realisation of the uncertainty. The lines in figure 4, are (inverse) residual demand curves for each scenario, and the dots on the left plot show the optimal dispatch points for each scenario. However, instead we must seek to maximise the expected profit, given the distribution of possible scenarios; the offer stack which maximises this expected profit is shown on the right of figure 4. Note that in order to comply with the monotonicity requirement, lower profits are attained for certain scenarios.

So how can we compute this offer stack? If we know in advance that the offer stack will cross the various scenarios in the following order: $\{\omega_1, \omega_2, \omega_3 ...\}$, then ensuring monotonicity could be simply achieved by adding linear constraints, as follows:

$$y^{\omega_i} \leq y^{\omega_{i+1}}, \quad i = 1... |\omega| - 1,$$
$$p^{\omega_i} \leq p^{\omega_{i+1}}, \quad i = 1... |\omega| - 1.$$

However, where is it not possible to predict the order of the scenarios, an integer programming approach is required. To do this we introduce a set of binary variables: w_{ij} which is set to 1 if scenario ω_j is passed through after scenario ω_i in the offer stack. Finally, note that each scenario $\omega \in \Omega$ has a corresponding probability of ρ^ω. Thus we can create the following profit maximization problem.

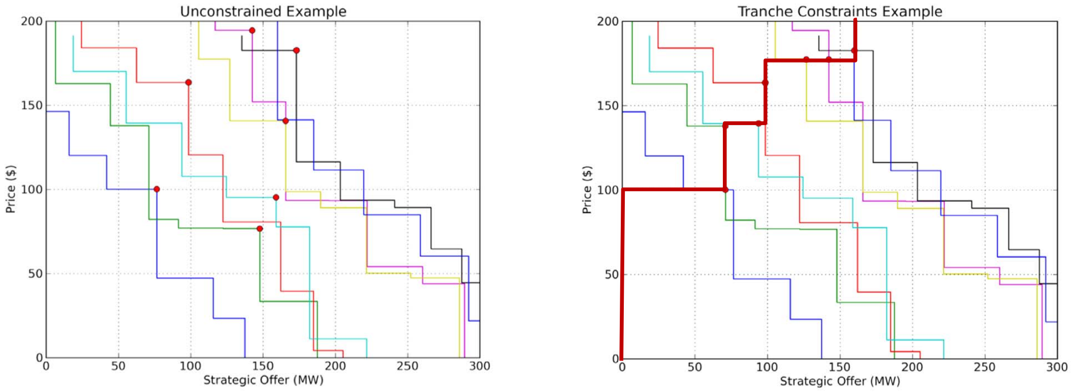

Figure 4: Optimal dispatch points (left); optimal offer stack (right).

Offer Model

$$
\begin{aligned}
\max \quad & \sum_{\omega \in \Omega} \left[d^{\omega} p^{\omega}(t^{\omega}) - p q^{\omega}(t^{\omega}) \right] & & \\
\text{s.t.} \quad & q^{\omega}(t) = d^{\omega} - y^{\omega} & \forall \omega \in \Omega & \\
& 0 \leq y^{\omega} \leq C & \forall \omega \in \Omega & \\
& y^{\omega_j} \leq y^{\omega_i} + M w_{ij} & \forall \omega_i \in \Omega, \omega_j \in \Omega, i \neq j & \quad (5) \\
& p^{\omega_j}(t^{\omega_j}) \leq p^{\omega_i}(t^{\omega_i}) + M w_{ij} & \forall \omega_i \in \Omega, \omega_j \in \Omega, i \neq j & \quad (6) \\
& w_{ij} + w_{ji} = 1 & \forall \omega_i \in \Omega, \omega_j \in \Omega, i \neq j & \quad (7) \\
& w_{ij} \in \{0, 1\} & \forall \omega_i \in \Omega, \omega_j \in \Omega, i \neq j. &
\end{aligned}
$$

Explanation

The objective now maximises expected revenue. The variable w_{ij} takes the value 1 if the RDC of scenario ω_i is intersected before that of scenario ω_j, by an increasing offer stack starting at the origin, and otherwise 0. Constraints (5) and (6) enforce this requirement, using a big-M formulation. Constraint (7) ensures that if ω_i comes before ω_j, then ω_j comes after ω_i.

4 Optimisation over a transmission network

In the previous section, we have been assuming that all offers and demand occur at a single node. In this section, we will detail how the generator offer model can be extended to the case where there is an underlying network. The dispatch problem in this case must be extended, to include transmissions lines and capacities. This is presented in the next section; following this we will show how the formulation extends to allow us to optimise the generation of multiple generators located over a network. The model that we have developed is called the *Offer Model over Electricity Networks* (OMEN).

4.1 Network dispatch problem

Here we will present the network dispatch model, which is solved by the system operator to compute how much power each generator is required to produce as well as the nodal prices. As in the single-node case, the optimality conditions of this linear program must be embedded within OMEN.

Sets and indices

$n \in \mathcal{N}$: the set of all nodes;
$ij \in \mathcal{A}$: the set of all arcs (lines);
$k \in \mathcal{L}$: the set of loops in the network;
$t \in \mathcal{T}_n$: the set of all tranches offered at node n,
$t \in \mathcal{T}$: the set of all offered tranches.

Parameters

q_t = offer quantity for tranche t;
p_t = offer price for tranche t;
d_n = demand at node n;
y_n = amount of power generated by strategic generator at node n;
K_{ij} = capacity of arc joining nodes i and j;
$L_{ij,k}$ = reactance of arc ij in loop k, its sign reflects its direction within the loop.

Decision variables

x_t = the dispatched quantity for tranche t;
π_n = the nodal price at node n.

Network Dispatch Model

$$\begin{aligned}
\min \quad & \sum_{t \in \mathcal{T}} p_t x_t \\
\text{s.t.} \quad & \sum_{t \in \mathcal{T}_n} x_t - \sum_{i|ni \in \mathcal{A}} f_{ni} + \sum_{i|in \in \mathcal{A}} f_{in} = d_n - y_n \qquad \forall n \in \mathcal{N} \quad [\pi_n] && (8) \\
& \sum_{ij \in \mathcal{A}} L_{k,ij} f_{ij} = 0 \qquad \forall k \in \mathcal{L} \quad [\lambda_k] && (9) \\
& -K \le f_{ij} \le K \qquad \forall ij \in \mathcal{A} \quad [\eta_{ij}^-, \eta_{ij}^+] && (10) \\
& 0 \le x_t \le q_t \qquad \forall t \in \mathcal{T}.
\end{aligned}$$

Explanation

Again, the objective is to minimise the total cost of dispatched tranches. However, constraint (8) now requires the generation, plus imports, less exports to equal the nodal demand at every node; the shadow price of each of the constraints is π_n, the nodal price. Constraint (9) ensures that the power flow around any loop complies with the physical laws. Finally, constraint (10) ensures that the flow on each line is less than the capacity of the line (in both directions).

4.2 OMEN: Offer Model over an Electricity Network

We will now show how we are able to embed the optimal full network dispatch within OMEN, without any bilinear terms. By deriving the optimality conditions of the network dispatch model, we have:

$$-\pi_i + \pi_j - \eta_{ij}^+ + \eta_{ij}^- + \sum_{k\in\mathcal{L}} \lambda_k L_{ij,k} = 0, \quad \forall ij \in \mathcal{A}, \tag{11}$$

$$0 \leq \eta_{ij}^+ \perp K_{ij} - f_{ij} \geq 0, \quad \forall ij \in \mathcal{A}, \tag{12}$$

$$0 \leq \eta_{ij}^- \perp K_{ij} + f_{ij} \geq 0, \quad \forall ij \in \mathcal{A}. \tag{13}$$

Now let us consider the objective that we would have in the network case, with potentially a generator at each node.

$$\begin{aligned}
\sum_{n\in\mathcal{N}} y_n \pi_n &= \sum_{n\in\mathcal{N}} \left(d_n - q_n(t_n) + \sum_{i|ni\in\mathcal{A}} f_{ni} - \sum_{i|in\in\mathcal{A}} f_{in} \right) p_n(t_n) \\
&= \sum_{n\in\mathcal{N}} \left[d_n p_n(t_n) - p q_n(t_n) \right] + \sum_{n\in\mathcal{N}} \left[\left(\sum_{i|ni\in\mathcal{A}} f_{ni} - \sum_{i|in\in\mathcal{A}} f_{in} \right) p_n(t_n) \right] \\
&= \sum_{n\in\mathcal{N}} \left[d_n p_n(t_n) - p q_n(t_n) \right] + \sum_{ij\in\mathcal{A}} \left[p_i(t_i) - p_j(t_j) \right] f_{ij}
\end{aligned}$$

The first summation is identical in form to the single node case, and is piecewise-linear, however the second summation is bilinear. To correct for this, consider the arc flow stationarity constraint, equation (11), which can be rearranged to isolate the difference in nodal prices at either end of each line:

$$p_j(t_j) - p_i(t_i) = \eta_{ij}^+ - \eta_{ij}^- - \sum_{k\in\mathcal{L}} \lambda_k L_{ij,k}, \quad \forall ij \in \mathcal{A}.$$

This is then substituted into the objective function, yielding:

$$\sum_{n\in\mathcal{N}} \left[d_n p_n(t_n) - p q_n(t_n) \right] - \sum_{ij\in\mathcal{A}} \left[\eta_{ij}^+ - \eta_{ij}^- - \sum_{k\in\mathcal{L}} \lambda_k L_{ij,k} \right] f_{ij}.$$

By applying Kirchoff's law in equation (9) this can be simplified to:

$$\sum_{n\in\mathcal{N}} \left[d_n p_n(t_n) - p q_n(t_n) \right] - \sum_{ij\in\mathcal{A}} \left[\eta_{ij}^+ - \eta_{ij}^- \right] f_{ij}.$$

Now consider the complementary slackness constraints (12) and (13). The instance when $\eta_{ij}^+ > 0$ can only occur when $f_{ij} = K_{ij}$; and similarly, $\eta_{ij}^- > 0$ can only occur when $f_{ij} = -K_{ij}$. When $\eta_{ij}^+ = \eta_{ij}^- = 0$, then $-K_{ij} \leq f_{ij} \leq K_{ij}$. All three scenarios can be represented by modifying the second term:

$$\sum_{n\in\mathcal{N}} \left[d_n p_n(t_n) - p q_n(t_n) \right] - \sum_{ij\in\mathcal{A}} |\eta_{ij}^+ - \eta_{ij}^-| K_{ij}.$$

Since the flow on a line cannot be binding at both its upper and lower limit, at most one of η_{ij}^+ and η_{ij}^- may be nonzero at a time. Because of this, the absolute value of the difference can be replaced by the sum:

$$\sum_{n\in\mathcal{N}} \left[d_n p_n(t_n) - p q_n(t_n) \right] - \sum_{ij\in\mathcal{A}} \left[\eta_{ij}^+ + \eta_{ij}^- \right] K_{ij}. \tag{14}$$

Finally, we must model the complementary slackness constraints using binary variables and the big-M method in order to formulate this as an integer program. Incorporating the new objective given in equation (14) and these new binary variables into the previous generator offer model forms OMEN.

OMEN

$$
\begin{aligned}
\max \quad & \sum_{n\in\mathcal{N}} [d_n p_n(t_n) - p q_n(t_n)] - \sum_{ij\in\mathcal{A}} \left[\eta_{ij}^{+} + \eta_{ij}^{-}\right] K_{ij} \\
\text{s.t.} \quad & q_n(t_n) = d_n - y_n & \forall n \in \mathcal{N} \\
& -p_i(t_i) + p_j(t_j) - \eta_{ij}^{+} + \eta_{ij}^{-} + \sum_{k\in\mathcal{L}} \lambda_k L_{ij,k} = 0 & \forall ij \in \mathcal{A} \\
& \sum_{ij\in\mathcal{A}} L_{k,ij} f_{ij} = 0 & \forall k \in \mathcal{L} \\
& 0 \le y_n \le C_n & \forall n \in \mathcal{N} \\
& 0 \le K_{ij} - f_{ij} \le 2K_{ij}\left(1 - z_{ij}^{+}\right) & \forall ij \in \mathcal{A} \quad (15) \\
& 0 \le K_{ij} + f_{ij} \le 2K_{ij}\left(1 - z_{ij}^{-}\right) & \forall ij \in \mathcal{A} \quad (16) \\
& 0 \le \eta_{ij}^{+} \le M z_{ij}^{+} & \forall ij \in \mathcal{A} \quad (17) \\
& 0 \le \eta_{ij}^{-} \le M z_{ij}^{-} & \forall ij \in \mathcal{A} \quad (18) \\
& z_{ij}^{+},\, z_{ij}^{-} \in \{0, 1\} & \forall ij \in \mathcal{A}. \quad (19)
\end{aligned}
$$

Explanation

For simplicity, we have formulated this for a single scenario (as opposed to the multiple scenario model shown earlier), however, the extension to multiple scenarios follows as before (see Weng, 2013 for the full formulation) . Equations (15)-(19) model the complementary slackness constraints using binary variables.

4.3 Two-node Example

To understand some of the incentives that firms have when offering at multiple nodes, let us first consider a simple two-node network (A–B), joined by a single line. The strategic firm has a plant at each node, with the plant at node A having slightly higher marginal costs. At each node there are a number of other non-strategic generators submitting offers, and what we wish to do is find a pair of offer stacks (one at each node) for the strategic firm that maximises its profit over both nodes. To understand the effect that a line can have, we will examine the optimal offer strategy under three line capacities: a capacity of 0MW, which is equivalent to two separate markets; an infinite capacity, which is equivalent to a single unified market; and a 50MW line, which may or may not become congested, depending on how the strategic firm offers. The final of these cases is the most interesting as the other two reduce to single-node problems.

Figure 5 shows the optimal offer stacks of the firm at node A (left) and node B (right). The line marked with a square is the optimal offer when the line is 0MW. We can see that the prices offered at the nodes are different, and power is withheld at both nodes; this is because the two markets are independent. At the other extreme, when the line is infinitely large (the offer stack marked with a triangle), we see that the optimal strategy is to fully utilise the cheaper generator (at node B), and withhold at node A in order to maintain a higher price (which is the same at both nodes). Interestingly, the optimal strategy for when the line is 50MW (marked with a circle), does not lie between these extremes. Instead, the firm withholds at node B and attempts to be fully dispatched at node A – this causes the transmission line between nodes A and B to become congested, with the price at node B higher than that of node A. This occurs because the residual demand elasticity is lower at node B meaning that prices increase more rapidly when withholding.

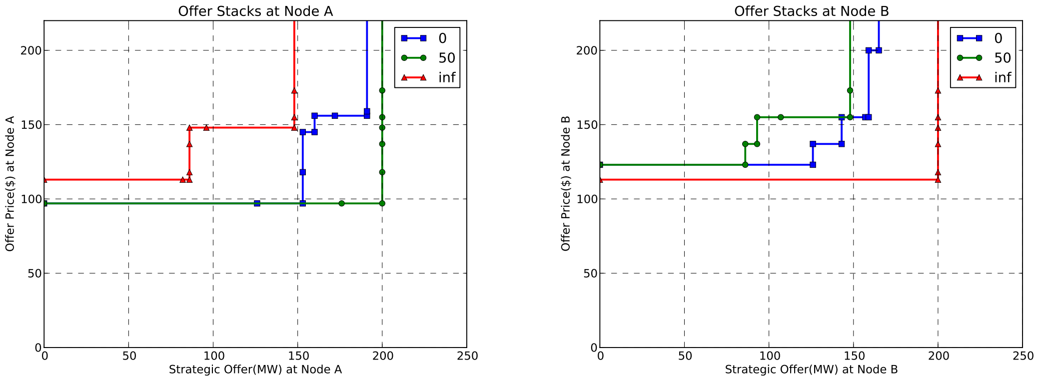

Figure 5: Optimal offer stacks.

This behaviour could not have been captured using traditional single-node models, since the line is not becoming congested in all scenarios, and thus the curves must be constructed simultaneously. Furthermore, understanding these strategic incentives that exist over networks is important not only for generators, by also for the regulator (in New Zealand, the Electricity Authority); this is because in order to have a well-functioning and efficient market, the correct incentives should be in place. Particulary, this may be useful when considering line upgrade proposals, since the change in incentives can be identified.

5 Extensions

The model currently is able to be run using real New Zealand generation and transmission data. However, this is not presented here due to space constraints. Nonconvex cost functions and contract positions for gentailers (generators/retailers) can also be incorporated. Transmission losses, mathematically can be included in the model, however, this significantly increases the size of the model needing to be solved. We are currently investigating methods to quickly find good incumbants to speed up the branch and bound process. Finally, we are intending to add the reserve market to this model, and consider how one might offer into the energy and reserve market simultaneously.

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