Shortest Path Algorithms in Traffic Assignment

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Abstract

Finding shortest paths is a classical well-studied problem in optimisation. Many solution algorithms for practical optimisation problems need to determine shortest paths as sub-tasks. An example of such a problem is Traffic Assignment (TA).

Various TA models are used in practice in order to: analyse the current usage of a road network; predict the impact of potential projects and policies; control traffic: level of congestion, emission, toll revenue etc (Ortúzar and Willumsen 2001). These models aim to predict how the road users will decide to travel during a given period of time (usually morning and evening peaks are of interest). The task of predicting how a particular individual will travel is not trivial and it is addressed by making some assumptions on how people usually make their route choices. The classical TA model assumes that every driver travels on their fastest (or shortest) path and network equilibrium occurs when no traveller can decrease their travel time by shifting to a new path. This assumption is called user equilibrium condition or Wardrop's first principle (Sheffi 1985).

The key feature of TA models consists in taking into account congestion effects that occur if capacities of some roads are exceeded. In order to consider congestion, so-called link cost functions are used. They represent a travel time through a given link of a network depending on the traffic flow on that particular link. Thus, given the transportation network, the number of drivers travelling, their origins and destinations and the link cost functions, the goal is to assign traffic flows to links of the road network in such a way that the user equilibrium condition is satisfied.

Conventional TA algorithms are iterative (Florian and Hearn 1995). They start with a feasible initial assignment of flows and proceed to the equilibrium solution by iteratively improving the current flow pattern. As a result link flows of the transportation network change after each such improvement and corresponding link travel times change as well. Usually in order to improve the current solution shortest path calculations are required. The number of such calculations depends on the particular TA algorithm used. This iterative nature together with the specific structure of the underlying transport networks raises the question which shortest path methods are most suitable for an iterative TA algorithm. Therefore, we conduct empirical tests of the performance of different shortest path approaches in the context of TA.

Many different TA algorithms were proposed in the literature. However, in the following we limit our study to path equilibration algorithm only. This TA method

decomposes the original problem by origin-destination (O-D) pairs (Florian and Hearn 1995). In particular it considers one O-D pair at a time, i.e. flows of all O-D pairs are assumed to be fixed except the O-D pair under consideration. This algorithm requires storing the sets of active paths (paths with positive flow). In order to prevent storing all possible paths for each O-D pair a *column generation* approach is usually applied, which consists in generating new paths when needed (Patriksson 1994). It is performed as follows: for a given O-D pair find the shortest path and add it to the set of active paths if it is shorter than the current shortest path contained in this set. Also, in order to keep only promising paths, the paths that do not carry flow are removed.

Therefore, at each iteration of column generation we need to find a shortest path. However, if during the previous iteration only a few link flows actually changed or if the flow change was not significant, the shortest path of the O-D pair under consideration might remain the same as before. Therefore, we might be able to avoid calculating it again. Two situations are possible if we decide to skip calculation of the shortest path for this O-D pair. If the shortest path remains the same between the previous and current iterations, we have successfully avoided the calculation and reduced the computational time. On the other hand, if the path changes, the TA algorithm may not move closer to equilibrium after the current iteration, which might cause an increase of the total number of iterations and computational time. Although the number of iterations may vary, a path equilibration algorithm will still converge if an appropriate shortest path avoiding strategy is used. Such a strategy must ensure that a new shortest path will be calculated eventually.

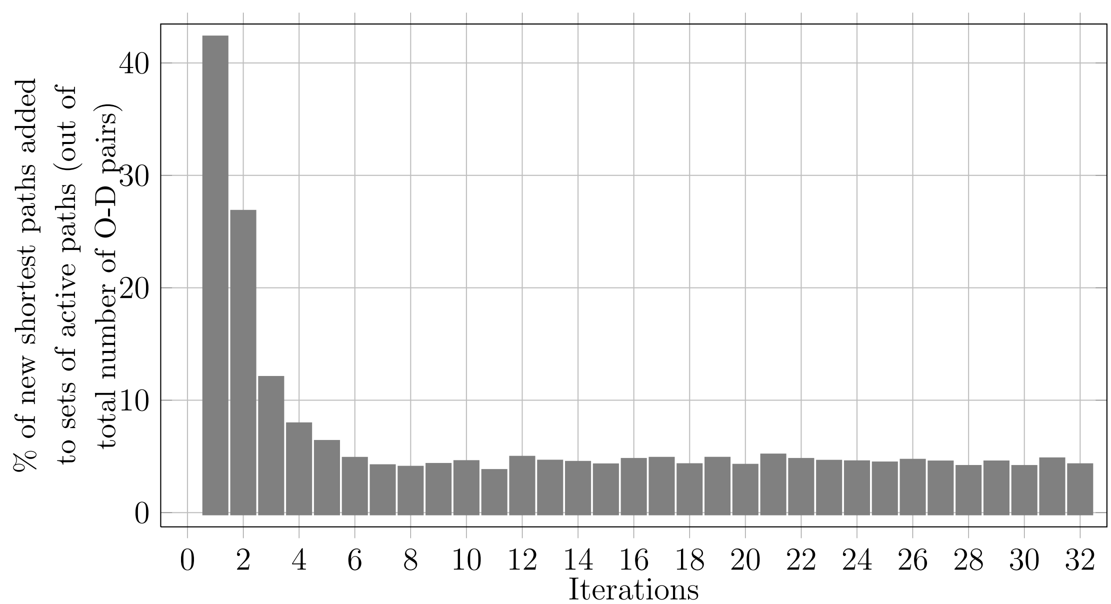

Figure 1: Shortest path calculations for ChicagoSketch instance.

For example, consider Figure 1 that shows the percentage of the number of shortest paths that actually changed during the initial iterations. As can be seen, after the 6$^{\text{th}}$ iteration new shortest paths were added to the active path sets of only 5% of all OD pairs. Based on this observation we investigate if avoiding some shortest path calculations altogether improves the overall runtime of TA algorithms.

We propose two strategies of avoiding shortest path calculations. The first one is

based on iterations. For each O-D pair, we check if its shortest path did not change in the previous two iterations, then we delay the next shortest path calculation by a few iterations. The second strategy consists in skipping shortest path calculations randomly. That is, when it is required to calculate the shortest path for a given O-D pair, we first generate a random number and then decide whether to perform the calculation based on that number.

In this work we conduct a numerical study of different shortest path algorithms in the context of TA and analyse several shortest path computation avoidance strategies. These ideas were applied to a path equilibration algorithm but can be extended to other TA methods as well.

Key words: Shortest path, traffic assignment, numerical study.

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