Binary Interruptible Load Optimisation

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Abstract

The electricity market in New Zealand relies on reserves to restore balance between supply and demand following a fault in the network. Interruptible load reserve is one type of reserve that responds either in full, or not at all. Currently these reserves can be partially dispatched, which may lead to an over-response of reserve, referred to as overhang. We investigate two models for reducing this overhang. The Intelligent Selection model constructs a dispatch solution that minimises the overhang without changing the cost of the initial solution. This eliminates the overhang in 15% of solutions. The Constrained On/Off model alters reserve dispatches more significantly to eliminate overhang from the solution entirely. Additional payments are made so that all reserve offers are fairly compensated for these changes. Overhang is eliminated in all solutions for a total cost increase of 0.4%. Finally, we consider the problem of determining the price in the new solutions.

Key words: electricity markets, reserve markets, interruptible load reserve.

1 Introduction

Security of electricity supply is a key concern of Transpower, the System Operator of the New Zealand electricity network. Decreases in the supply frequency from its normal level of 50 Hz pose the biggest threat to grid stability, and occur when demand exceeds supply. When the frequency falls below 47 Hz, a cascading failure of generators occurs, and the entire grid goes offline, at great cost.

The market procures reserve to guard against failures of this nature. In the event of a generator failure and the following decrease in supply, reserves act to restore the balance between supply and demand. This is done either by increasing generation output (known as spinning reserve), or by shedding demand (known as interruptible load reserve). Procured reserves act sufficiently fast such that the frequency is prevented from falling below 48 Hz. Any response of this nature defines an under-frequency event. Historically, 10–12 such events have occurred annually.

Similarly, if the frequency increases beyond 52–53 Hz as the result of supply exceeding demand, an over-frequency event occurs. This is less damaging than an under-frequency event, but still constitutes a security concern.

Of particular interest in this paper is interruptible load reserve (ILR), which represents consumers shedding load from the grid in case of an under-frequency event. By their nature, these reserves must respond with their full offer if called upon

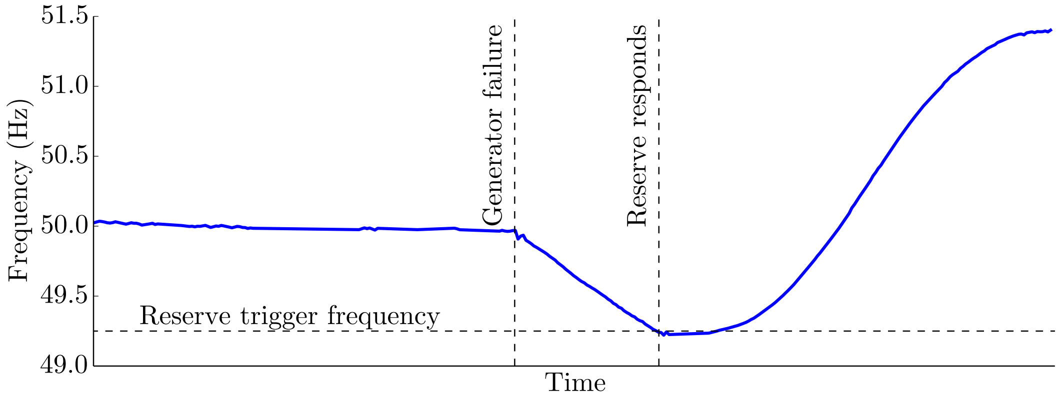

Figure 1: Example of under-frequency event with a reserve over-response.

to respond, unlike other reserve types which can respond in continuous amounts. This restriction is not observed by the Scheduling, Pricing and Dispatch (SPD) optimisation engine that determines dispatch in the network; this can result in a reserve solution containing ILR that is partially dispatched. If an under-frequency event were to occur, this ILR would have to respond with its full offer tranche, leading to a reserve response greater than required. The grid thus experiences an over-response of reserve, which causes supply to exceed demand. This excess leads to an increased frequency on the grid, and if the over-response is large enough, can even cause an over-frequency event. An example of reserve over-response and the effect on frequency is shown in Figure 1.

We investigate methods of minimising the amount of reserve over-response, termed *overhang*, that is present in reserve market dispatch solutions, such that the risk of an under-frequency event transitioning into an over-frequency event is minimised.

2 Current operation of the reserve market

2.1 Formulation of current operation

The dispatch of reserves is carried out within SPD. Participants submit reserve offers into the market with a price and quantity, and the market is cleared at least cost, with all dispatched reserve being paid the market clearing price.

In order to clear the market in this way, the reserve is dispatched in merit order. This is equivalent to sorting the reserve offers by increasing price, and forming an aggregate offer stack for this reserve market. The offers are then accepted in this order, starting with the lowest price offer, until enough reserve has been procured. The clearing price is then equal to the marginal price of reserve, which is the highest offer price among those reserve offers that are dispatched. An example of a dispatch solution is shown in Figure 2.

Dispatching the reserve offers in merit order is equivalent to minimising the total cost of dispatched reserve, where the cost of each reserve is its offer price. This is achieved by solving the following LP:

Indices

t = reserve offer tranche: $T = \{1, 2, \dots, n\}$.

Parameters

w_t = reserve quantity (MW) offered by tranche t;

p_t = offer price (\$/MW) for tranche t;
b = total reserve quantity required.

Decision variables

x_t = proportional dispatch of tranche t.

Model ReserveClearingCurrent

$$\begin{aligned} \min \quad & \sum_{t \in T} p_t w_t x_t & & \\ \text{s/t} \quad & \sum_{t \in T} w_t x_t \geq b & [\pi \geq 0] \quad & (1) \\ & 0 \leq x_t \leq 1 \qquad \forall t \in T. & & (2) \end{aligned}$$

Explanation

The objective is to minimise total reserve procurement cost. Constraint (1) ensures the procured reserve satisfies demand. Constraint (2) ensures each tranche is dispatched between 0 (no dispatch) and 1 (complete dispatch). The clearing price in the market is given by the dual variable, π.

This model deals with the reserve market in isolation, which is not the case in SPD, where the energy and reserve markets are co-optimised. To achieve this, the co-optimisation problem is first solved, and then the generation results are fixed, allowing the reserve market to be solved separately. This simplified model allows easier and more direct analysis of the reserve market problem. We believe that this modification does not greatly impact the results, due to the reserve market being substantially smaller than the energy market, and hence the reserve results changing should have little effect on the optimal generation solution. We are also confident that the methods we propose would be able to be implemented without issue within SPD.

2.2 Issues with current operation

Recall that ILR can only respond in full or not at all. The corresponding dispatch variables should hence be binary rather than continuous, but this is not the case under current operation. We are interested in the level of overhang in the solution,

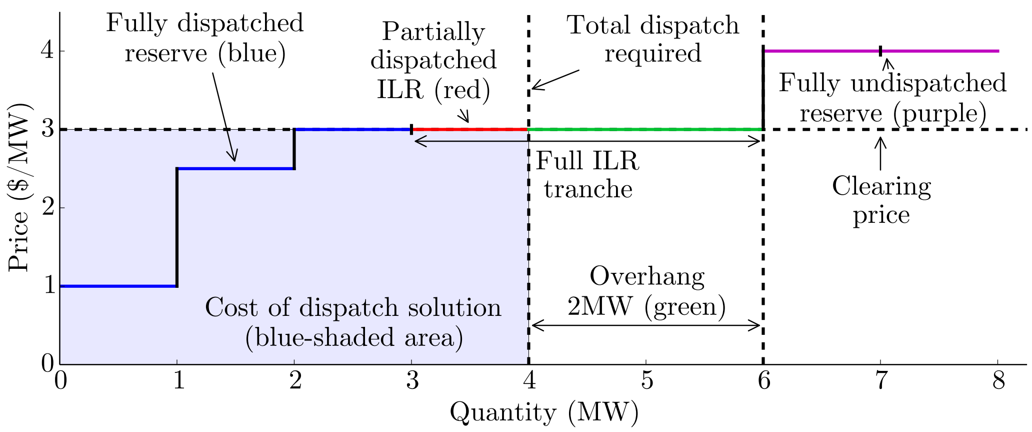

Figure 2: Example of reserve dispatch solution containing overhang.

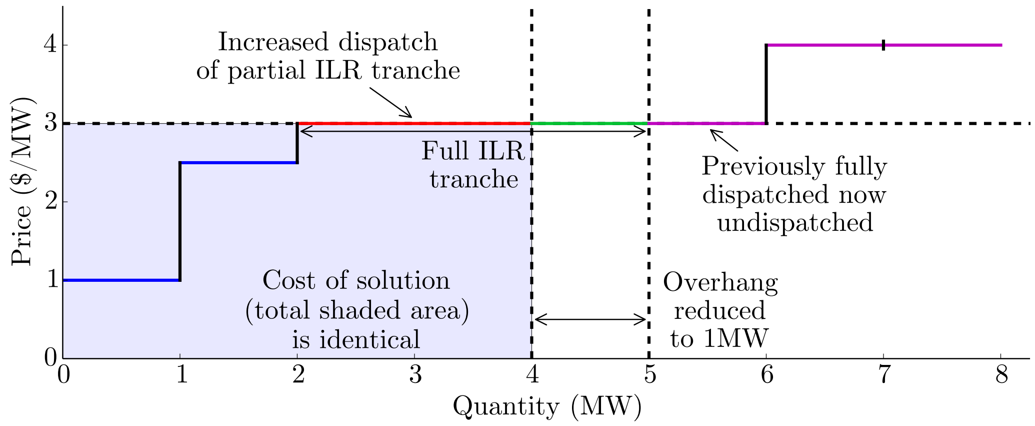

Figure 3: Example of reserve dispatch solution using intelligent selection.

which is given by the total difference between the response and the dispatch of ILR offers. Since this is a minimisation knapsack problem, there can be at most one fractional variable in the LP solution, meaning that only one ILR offer can be partially dispatched and lead to overhang.

In this following sections, we will investigate ways of efficiently reducing the level of overhang in solutions to ensure that the risk of experiencing an over-frequency event is minimised.

3 Intelligent selection method

When there are multiple reserve offer tranches at the clearing price, there can be multiple optimal solutions to the model ReserveClearingCurrent. The dispatches of these clearing tranches can be changed freely without altering optimality as long as the total dispatch remains the same, because each of these tranches has the same price. In this situation, SPD will use the first optimal basis that is found.

It is possible that these equivalent solutions may have differing levels of overhang. In this case, we wish to select the solution with the smallest overhang. This solution will still be optimal for the original reserve market problem, and so the cost of procurement will be identical – we are simply selecting the "best" optimal solution. This forms the motivation for the *intelligent selection method*.

3.1 Implementation of intelligent selection method

In order to implement this method, we take an approach that minimises the overhang by altering the initial optimal solution to the reserve market problem. This is done by fixing the dispatch of all tranches above and below the clearing price, π, and then using a knapsack formulation to minimise the level of overhang.

In order for the solution to account for overhang, we restrict the dispatch variables of ILR to be binary, which accounts for the all-or-nothing nature of their response.

The MIP formulation for the intelligent selection model is as follows:

Parameters

π = clearing price in solution to ReserveClearingCurrent;
$T_\pi^+ = \{t \in T \mid p_t > \pi\}$;
$T_\pi^- = \{t \in T \mid p_t < \pi\}$;
$T_I = \{t \in T \mid \text{tranche } t \text{ is ILR}\}$.

Model ReserveClearingIntelligent

$$
\begin{aligned}
\min \quad & \sum_{t \in T} w_t x_t - b \\
\text{s/t} \quad & \sum_{t \in T} w_t x_t \geq b \\
& x_t = 0 \qquad \forall t \in T_\pi^+ \qquad (3) \\
& x_t = 1 \qquad \forall t \in T_\pi^- \qquad (4) \\
& 0 \leq x_t \leq 1 \qquad \forall t \in T \\
& x_t \in \{0, 1\} \qquad \forall t \in T_I. \qquad (5)
\end{aligned}
$$

Explanation

The objective is to minimise the overhang in the solution, which is given by the difference between the total reserve response and the total dispatch. The objective value is clearly non-negative, and if the optimal solution has objective value zero, the overhang will have been eliminated entirely.

Constraints (3) and (4) fix the dispatch of the tranches above and below the clearing price respectively. Constraint (5) ensures that ILR tranches can only respond in full if responding. The remaining constraints are identical to the previous model.

An example of applying this method to the previous example is shown in Figure 3.

3.2 Application of intelligent selection to market data

In order to evaluate the effects of the Intelligent Selection method, it was tested on historical market data. Each reserve market was solved twice, once using Reserve-ClearingCurrent to simulate the SPD solution, and then once using the intelligent selection model with π set to be the clearing price from the SPD solution. This was repeated for all data from January 2013 to August 2013.

Of the 46,082 problems solved using SPD, overhang was present in 9,059 solutions, or 19.7 % of cases. Of these 9,059 cases with overhang, the Intelligent Selection method was able to eliminate the overhang in 1380 solutions, or 15.2 % of the time. There was a reduced level of overhang present in 472 solutions, or 5.2 % of cases.

It is strongly recommended that such a method is adopted by Transpower in the solution of the reserve market problem as a bare minimum, since these reductions in overhang come at no additional procurement cost to the system, due to the solutions produced still being optimal for the original problem.

The Intelligent Selection method is unable to eliminate the overhang in 84.8 % of solutions with overhang. This means that 16.7 % of solutions overall will still have overhang present after being solved using the Intelligent Selection method. This is a significant proportion of reserve market problems that remain at risk of reserve over-response due to overhang, so methods allowing further reduction of overhang in these cases need to be devised.

4 Constrained on/off payment method

The intelligent selection method reduces the overhang as much as possible without incurring any additional cost. We saw that the overhang could be eliminated at no cost in 15.2 % of solutions with overhang present. However, the remaining 84.8 % of solutions still contained overhang. This motivates the need for a method that can reduce the overhang in these remaining solutions. This requires a method that reduces the overhang at some additional cost, since all reduction for no cost has already been attempted by intelligent selection.

4.1 Definition of constrained on/off actions

We introduce two types of action, each of which alters the overhang in a different way.

Constrained off tranches

We can reduce the dispatch of an offer that is currently dispatched. This allows us to increase the dispatch of the partially dispatched ILR tranche by an equal amount, which will reduce the total overhang in the reserve solution.

Each offer that is constrained off in this fashion will receive a payment. This payment is equal to the difference between the tranche offer price and the clearing price, multiplied by the reduction in dispatch. This compensates the participant for the profits they would have made had they been dispatched under business as usual.

Constrained on tranches

We can also increase the dispatch of an offer that is not yet fully dispatched. This allows us to reduce the dispatch of the partially dispatched ILR tranche by an equal amount. If the dispatch of this overhanging tranche can be reduced to zero, the overhang will vanish entirely as the tranche will no longer be utilised at all.

In order to ensure these offers do not incur a loss as a result of being constrained on, they receive extra compensation for their action. The extra payment is equal to the difference between the offer price and the clearing price multiplied by the change in reserve dispatch, just as for constrained-off payments.

Note that tranches at the clearing price are ineligible for constrained on/off payments since the difference between their offer price and the clearing price is zero, and hence the payment will also be zero.

4.2 Using constrained on/off payments to reduce overhang

It is possible to change the level of overhang in the dispatch solution by altering the dispatch and applying the appropriate constrained on/off payments. We can therefore modify the optimal dispatch solution from Model ReserveClearingIntelligent to reduce the overhang at least cost. To do this, we need to determine the optimal combination of constrained on/off payments to make. There are two ways to reduce the overhang:

- increase the dispatch of the partially dispatched ILR tranche, which reduces the difference between its dispatch and response amounts;
- decrease the dispatch of the partially dispatched ILR tranche to the point where it is undispatched, as then dispatch and response will both be zero.

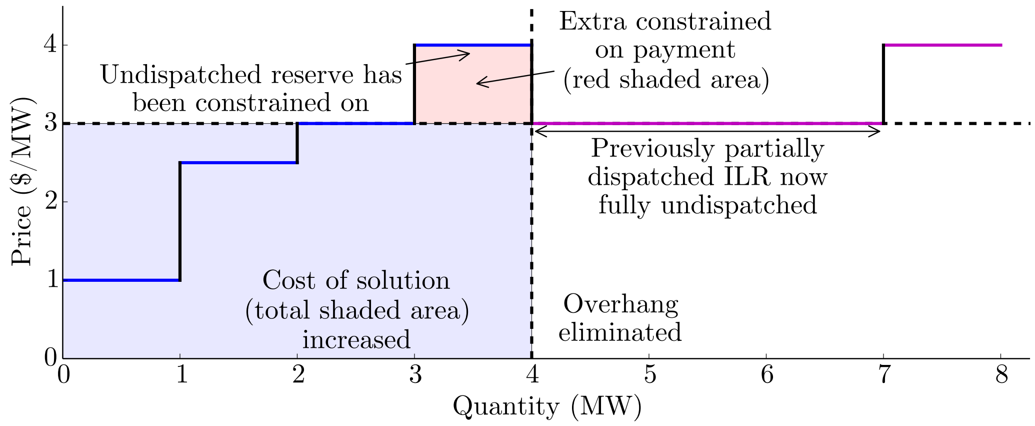

Figure 4: Example of reserve dispatch solution using constrained on payment.

We must select one of these approaches when reducing the overhang. This leads to the following formulation.

Parameters

i = index of the partially dispatched ILR tranche in ReserveClearingIntelligent;
$\hat{b}_o$ = level of overhang in solution to ReserveClearingIntelligent;
$\hat{b}_u$ = partial dispatch of tranche i in solution to ReserveClearingIntelligent;
x_t = proportional response of tranche t as solved in ReserveClearingIntelligent;
ϵ = maximum level of overhang allowed.

Decision variables

y_t = proportional response of tranche $t \in T \setminus \{i\}$;
z = binary variable determining whether dispatch of tranche i increases or decreases.

Model ReserveClearingPayments

$$\min \quad \sum_{\substack{t\in T\\ t\neq i}} (p_t - \pi) w_t (y_t - x_t)$$

$$\text{s/t} \quad \hat{b} \leq \sum_{\substack{t\in T\\ t\neq i}} w_t (y_t - x_t) \leq \hat{b} + \epsilon \qquad (6)$$

$$\hat{b} = z\hat{b}_u + (1 - z)(-\hat{b}_o) \qquad (7)$$

$$0 \leq y_t \leq 1 \qquad \forall t \in T \setminus \{i\}$$

$$y_t \in \{0, 1\} \qquad \forall t \in T_I \setminus \{i\}$$

$$z \in \{0, 1\}.$$

Explanation

The objective is to minimise the total cost of constrained on/off payments. The constrained on/off payment made to any tranche $t \neq i$ is given by the product of the change in dispatch and the difference between the offer price and the clearing price.

Constraint (7) sets the variable $\hat{b}$ to either $\hat{b}_u$ or $-\hat{b}_o$ in line with the value of z, representing the required change in reserve needed in either case. If we are increasing the dispatch of tranche i, the reserve response of all other tranches

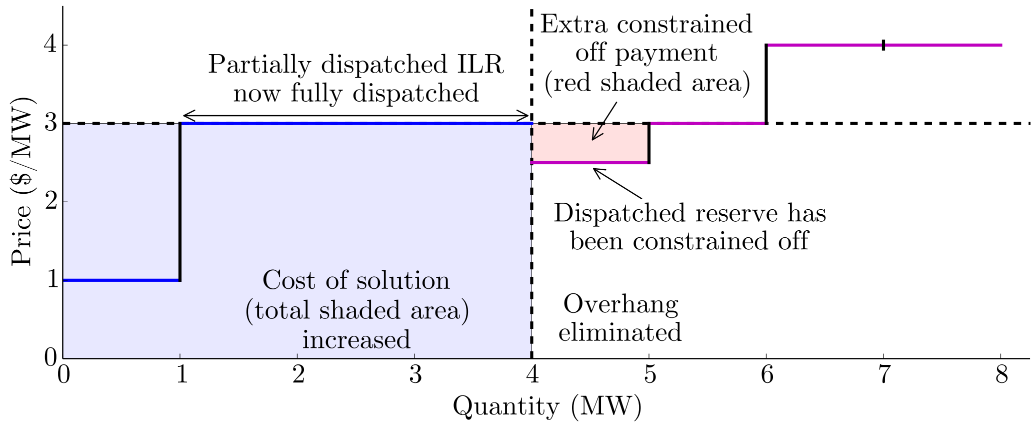

Figure 5: Example of reserve dispatch solution using constrained off payment.

needs to be reduced by between $\hat{b}_o - \epsilon$ and $\hat{b}_o$. If we are decreasing the dispatch of tranche i, the increase in reserve response of all other tranches needs to be between $\hat{b}_u$ and $\hat{b}_u + \epsilon$. Constraint (6) ensures that the total change in reserve response of all tranches $t \neq i$ is within these bounds.

Figures 4 and 5 show the previous example market solved using constrained on and off payments respectively.

5 Applying constrained on/off payment method to market

The total cost of procuring reserve across all of the periods processed was \$29.1 million when using ReserveClearingIntelligent. Any solutions that still contained overhang were further processed using ReserveClearingPayments to reduce the overhang below varying thresholds. There were around 8000 solutions that required this further processing, so we will examine these solutions to analyse the performance of the constrained on/off payment method. The reserve procurement cost across just these periods totalled \$9.76 million when using ReserveClearingIntelligent.

5.1 Elimination of Overhang

First we consider the effects of forcing the overhang to be eliminated entirely, by setting $\epsilon = 0$. Running the ReserveClearingPayments model on all periods with overhang remaining results in a total additional cost of just over \$115,000 to eliminate this overhang entirely.

This represents an increase in total procurement cost of 0.39 %, or an increase of 1.2 % in the total procurement cost when only considering periods with overhang.

5.2 Reduction of overhang to varying thresholds

Now we consider reducing the overhang to varying thresholds to investigate how the cost of reducing overhang varies with threshold. Table 1 shows the average cost per period of reducing the overhang below the given threshold, normalised against the level of reduction to give \$/MW. It was found that the cost of reduction depends heavily on the initial level of overhang, so the results are grouped by the amount of overhang present in the solution after solution with ReserveClearingIntelligent. There are a number of things we can draw from these results.

Threshold (MW)	Overhang from ReserveClearingIntelligent (MW)									
	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
0	0.93	0.68	0.70	0.75	0.65	0.47	1.78	1.68	6.87	2.68
10		0.83	0.74	0.70	0.61	0.48	1.79	1.68	7.17	2.64
20			0.85	0.71	0.55	0.48	1.83	1.68	7.43	2.60
30				0.84	0.53	0.49	1.80	1.69	7.68	2.56
40					0.56	0.55	1.71	1.74	7.43	2.51
50						0.66	1.61	1.90	7.12	2.44
60							1.62	2.13	6.69	2.41
Period Count	2702	1921	1347	674	486	453	56	24	10	6

Table 1: Average cost ($ per MW) of reducing overhang to given threshold.

The per-MW cost generally decreases as the size of the overhang increases and as the threshold decreases. We might perhaps have expected the opposite result; that it would become more expensive, per-MW, to reduce the overhang by a larger amount. This is because as more reduction is needed, the tranches being constrained on/off are likely to be further from the clearing price, thus increasing the marginal cost of reduction. The reason we do not see this is because of solutions with reserve that is constrained on. In the case of reserve being constrained on, the end dispatch is always zero, regardless of the threshold required. Thus once the constrained off solution is optimal, the average cost per MW of reduction does not increase with the amount of reduction required, instead it decreases, as greater MW reductions are achieved for the same cost.

We also see there is a sharp increase in the cost of reduction once the overhang passes 60 MW. A possible explanation for this sharp increase is that the solutions with high overhang are not independent of each other, and actually represent a very small set of data points, making their significance questionable. The required amount of reserve and reserve offer stack do not generally change much period to period, meaning a solution with large overhang may persist across multiple periods.

Investigating the solutions with overhang greater than 60 MW reveals this trend to be true. In nearly every case where large overhang occurred, it was present for at least three consecutive periods. This brings into question the independence of the 96 solutions we have with large overhang, and suggests there might be far fewer independent results in this set. Due to the very small size of the sample already, it is quite possible that the very large costs we are experiencing for these solutions are not representative of the true cost of reducing high levels of overhang.

6 Pricing in the new solutions

In the presented methods for reducing the overhang, we have assumed the clearing price remains unchanged in the solution to the constrained on/off problem. In the linear model, the clearing price is given by the dual variable π. However, the traditional concept of duality does not extend to mixed-integer program, and so the clearing price is not easily obtainable in this way.

The question of determining the best clearing price to use in the new solution is discussed in detail in Dunn (2013a). Two alternative methods of determining the clearing price are presented.

The first method fixes the reserve dispatches and finds the clearing price that minimises the sum of constrained on/off payments. The clearing price from this method is proven to coincide with the clearing price found in the original problem.

The second method uses integer duality to find the clearing price in the new solution. The method follows the approach presented by Gomory and Baumol (1960) and refined by Williams (1997), and is used to prove that the integer dual coincides with the original clearing price under certain assumptions.

One step in this second approach requires reducing the feasible region of our knapsack problem to the convex hull of feasible integer points. To do this, the algorithm detailed by Balas and Zemel (1984) was implemented. This procedure finds all facet-defining constraints of any 0-1 mixed-integer knapsack, and a software implementation of this algorithm has been made freely available (Dunn 2013b).

7 Conclusions

Following a generator failure, overhang poses a threat to grid security through the risk of over-response causing an over-frequency event. Two methods were investigated to reduce the overhang present in reserve dispatch solutions.

The intelligent selection method reduces overhang at no cost by selecting from the optimal solutions to the original problem that with the smallest overhang. This is able to eliminate the overhang in 15.2 % of cases, and can be implemented at no increase in cost, thus is strongly recommended.

The constrained on/off method uses additional payments to reduce the overhang even further. The overhang can be eliminated at a total increase in procurement cost of 0.39 %.

These models have been implemented outside of SPD, but we believe it is feasible to incorporate these changes into the SPD model, allowing these presented reductions to be realised in practice.

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