Modelling risk averse behaviour in electricity generation expansion

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Abstract

The current state of the art model for predicting expansion decisions is the Generation Expansion Model (GEM) which given many constraints, chooses to invest to maximize the welfare of the system. GEM does not model the uncertainty and the resulting risk averse behavior of the agents who make investment decisions.

In this project, a mathematical program was written to model the behavior of the various agents in the electricity market. Assuming all agents are perfectly competitive and risk neutral, this causes the model to give the same solution as the model which maximizes the welfare of the system. By adding a coherent risk measure, Conditional Value at Risk (CVaR) to the decision of expansion, we were able to present a more realistic representation of the market.

We compared various models using this this risk measure. We observed that in our example, eliminating the vertical integration between the generation and retail wing causes generation companies to under invest compared with the expected welfare maximising model. Including this vertical integration causes over investment compared with the expected welfare maximising model. We observe that this is the result of differences in profits between the two models across each of the scenarios.

1 Introduction

1.1 The Generation Expansion Model

GEM is a Mixed Integer Program (MIP) coded using GAMS (General Algebraic Modeling System), solved using the CPLEX solver (Bishop and Bull 2008). GEM uses the objective of minimizing the overall cost of investment, generation, transmission, use of reserve generation, and so forth. Solved under varying scenarios and subject to many constraints, which ensure the network is robust, the model essentially takes the cheapest overall plant from the stack and builds it when it is required or beneficial to the network.

1.2 Weaknesses of GEM

Using GEM to predict the construction and expansion of generation plants would be a reasonable approach if electricity generation were run centrally, but this is not the case in New Zealand. Since 1987, New Zealand has undergone a step-by-step process of industry reform which has changed the once centrally run system of providers of electricity generation, transmission, distribution and retailing, to create a competitive market in electricity generation and retailing (Evans 2005).

There are now 5 major Generation companies in New Zealand: Meridian Energy, Genesis Power, Mighty River Power, Contact Energy, and TrustPower. These *gen*eration companies each own their own re*tail* wing thus are commonly known as "gentailers". Due to this vertical integration across the electricity market, there is a different behavior to GEM, where each major gentailer primarily invests to keep their retail portfolios balanced.

1.3 Risk Aversion

The risk measure we will be using in our model, is the Conditional Value at Risk (CVaR). This is a coherent risk measure defined to equal the expected disbenefit (Z representing a random disbenefit) in the worst α scenarios. It can be represented by the well documented equation below (Philpott, Ferris, and Wets 2013):

$$\text{CVaR}_{1-\alpha} = \inf_t \left[t + \frac{1}{\alpha}\mathbb{E}(Z - t)_+ \right] \tag{1}$$

2 Problem formulation

In this section we formulate the equilibrium problem describing interaction between each of the agents. there are two levels to this problem, in the top level gentailers choose their expansion decisions to maximise their risk adjusted profit across all potential scenarios in the bottom level. In the bottom level we describe the interaction between the gentailers, the transmission operator, and the consumers of electricity.

2.1 Set, parameter, and variable definition

Sets

$\mathcal{G}$ = Generation firms ($g \in \mathcal{G}$);
$\mathcal{C}$ = Industrial Consumers ($c \in \mathcal{C}$);
$\mathcal{P}$ = All current and potential generation plants ($p \in \mathcal{P}$);
EXIST = Represents whether something currently exists or not (1 if it exists, 0 otherwise);
$\mathcal{K}$ = Types of generation plants ($k \in \mathcal{K}$);
$\mathcal{F}$ = Types of fuel used by generation plants ($f \in \mathcal{F}$);
$\mathcal{R}$ = Regions/nodes in the network ($r \in \mathcal{R}$);
$\mathcal{L}$ = Electrical lines in the network ($l \in \mathcal{L}$);
YEAR = Years modeled - Models long term trends ($yr \in YEAR$);
QTR = Quarters - Model seasonal variation ($q \in$ QTR);
LB = Load blocks - Models daily variations ($lb \in$ LB);
Ω := Possible realizations - Models uncertainty ($\omega \in \Omega$);

Mapping Sets

P_G: $\mathcal{P} \to \mathcal{G}$; P_R: $\mathcal{P} \to \mathcal{R}$; P_EX: $\mathcal{P} \to EXIST$;
P_PT: $\mathcal{P} \to \mathcal{K}$; PT_F: $\mathcal{K} \to \mathcal{F}$; I_R: $\mathcal{C} \to \mathcal{R}$;

Parameters

c_cap(p) = Cost of expanding generation plant p ($\$/MWs$);
c_fix(p) = Annual fixed cost of generation plant p per unit of capacity ($\$/MWyr$);
c_marg(p) = Marginal cost of generation for plant p ($\$/MWs$);
c_deliv(p) = The cost of delivering fuel to plant p ($\$/MJ$);
c_fuel(f, yr) = The projected cost of fuel type f during yr ($\$/MJ$);

p_ret = Retail price of electricity($\$/MJ$);
VOLL = Price of electricity which where retail demand becomes curtailed($\$/MJ$);
p_ind(c) = The price at which industrial consumer c curtails demand ($(\$/MW)/s$);

d_cap(p) =upper bound on the expansion of plant p (MW);
d_fof(p, ω) = Forced outage factor (forces a limit on the actual generation) ;
d_fom(k, lb, ω) = Factor modifying the effective capacity of generation plants of type k according to how they typically operate during load block lb, ω;
d_ret(yr, q, lb, ω, r) = The projected retail in yr, q, lb, ω, at node r (MW);
d_por(g, r) = Proportion of retail that industrial company g owns at node r;
d_ind(c) = The capacity of the industrial consumer c (MW);

t_yrs = The number of years modeled (yr);
t_hr2sec = Number of seconds in an hour (3600s/hr);
t_hrsInLB(y, q, lb) = Number of hours spent in load block lb during yr, q (hr);
t_prob(ω) = The probability of the realization of ω;

n_arc(r, l) := The node arc incidence matrix of the network;
n_cap(l) := Maximum amount of electricity which can be sent through line l (MW);
n_loss(l) := The quadratic loss factor across line l (proportional to the reactance of the line);

α := The Risk level;
λ := The weighting placed on the conditional Value at Risk;

Decision Variables

$x(p)$:= The expansion of plant p;
$y(p, yr, q, lb, \omega)$:= The level of generation from plant p during yr, q, lb, ω;
$gDB(g, yr, q, lb, \omega)$:= The negative profit per second obtained by gentailer g during yr, q, lb, ω;
$t(g, yr, q, lb)$:= Variable used obtain the Conditional Value at Risk (and will be equal to the Value at Risk) for gentailer g, yr, q, lb;
$gDB^+(g, yr, q, l, \omega)$:= A variable which is equal to the positive part of $Z - t$ (in equation (1)) and is zero otherwise $(Z - t)_+$ for gentailer g during yr, q, lb, ω;
$ttlGenDB(g)$:= The (risk adjusted) expected negative profit for gentailer g

across the entire period;

$\delta\_ret(r, yr, q, lb, \omega)$:= The amount of demand curtailed by the retail consumers at node r during yr, q, lb, ω (MW);
$conDisben(yr, q, \omega)$:= The negative welfare of all consumers during yr, q, lb, ω (\$);

$\delta\_ind(c, yr, q, lb, \omega)$:= The amount of demand curtailed by industrial consumer c during yr, q, lb, ω (MW);
$indDB(c, yr, q, lb, \omega)$:= The negative welfare the industrial consumer during yr, q, lb, ω (\$);

$f^{+}(l, yr, q, lb, \omega)$:= Positive part of the flow across line l during yr, q, lb, ω (MW);
$f^{-}(l, yr, q, lb, \omega)$:= Negative part of the flow across line l during yr, q, lb, ω (MW);
$\theta(r, yr, q, lb, \omega)$:= Voltage angle at node r during yr, q, lb, ω
$loss(r, yr, q, lb, \omega)$:= Loss of power at node r due to the flows along connecting lines during yr, q, lb, ω (MW);
$isoDB(yr, q, lb, \omega)$:= The negative profit gained by the ISO due to it's transmission during yr, q, lb, ω (\$);
$p(r, yr, q, lb, \omega)$:= The nodal price at node r during yr, q, lb, ω $(\$/MW)$;

2.2 Risk averse competitive equilibrium game

Here we present the objectives and constraints each agent is subject to. Using the EMP framework (GAMS Development Corporation 2009), we can formulate each problem by stating each agent's objective, decision variables, and corresponding constraints.

Gentailer objectives and constraints

The objective for the overall disbenefit for gentailer g has a weighting on the CVaR of the disbenefit.

$$\begin{aligned} ttlGenDB(g) = {} & \text{CostOfInv}(g) \\ & + (1-\lambda) \cdot \text{NegExpProf}(g) \\ & + \lambda \cdot \text{CVaRAlpha}(g) \qquad \forall g \in \mathcal{G} \end{aligned} \tag{2}$$

With the cost of investment including both the initial cost of investment and the operation and maintenance over the entire period.

$$\text{CostOfInv}(g) = \sum_{\substack{p \in \mathcal{P}: \\ P\_G(p)=g, P\_EX(p)=0}} \Big[(\text{t\_yrs} \cdot \text{c\_fix(p)} + \text{c\_cap(p)})\, x(p) \Big] \tag{3}$$

Equation (3) gives the cost gentailer g incurs running each generation plant p minus the fixed short run marginal costs over the period.

Constraints (4) and (5) define the lower and upper bound of expansion of each type of plant.

$$0 \leq x(p) \leq \text{d\_cap}(p) \qquad \forall p \in \mathcal{P} : P\_EX(p) = 0 \tag{4}$$
$$x(p) = \text{d\_cap}(p) \qquad \forall p \in \mathcal{P} : P\_EX(p) = 1 \tag{5}$$

$$\text{NegExpProf}(g) =$$
$$\text{t\_hr2sec} \sum_{\substack{yr \in YEAR,\\ q \in QTR,\\ lb \in \text{LB}}} \left[\text{t\_hrsInLB}(y, q, lb) \cdot \sum_{\omega \in \Omega} \left(\text{t\_prob}(\omega) \cdot gDB(g, yr, q, lb, \omega) \right) \right] \tag{6}$$

Equation (6) is the negative of the expected short run profit summed up over the entire period.

$$\text{CVaRAlpha}(g) =$$
$$\text{t\_hr2sec} \cdot \sum_{\substack{yr \in YEAR,\\ q \in QTR,\\ lb \in \text{LB}}} \left[\text{t\_hrsInLB}(y, q, lb) \cdot CVaR_{1-\alpha}(gDB(g, yr, q, lb, *)) \right] \tag{7}$$

Equation (7) is the negative of the expected short run profit in the worst α scenarios.

$$CVaR_{1-\alpha}(gDB(g, yr, q, lb, *)) = t(g, yr, q, lb) + \frac{1}{\alpha} \mathbb{E}_{\omega}(gDB(g, yr, q, lb, \omega) - t(g, yr, q, lb))_{+}$$
$$= t(g, yr, q, lb) + \frac{1}{\alpha} \sum_{\omega \in \Omega} \left(\text{t\_prob}(\omega) \cdot gDB^{+}(g, yr, q, \omega) \right) \tag{8}$$

Using equation (1), the CVaR is determined by making the substitution in equation (8). Here the optimal value for $t(g, yr, q, lb)$ (which is equal to the value at risk, at the risk level α) is found naturally as the agent attempts to minimize their risk weighted overall disbenefit.

Conditional Value At Risk constraints (yr, q, lb, ω)

$$gDB^{+}(g, yr, q, \omega) \geq gDB(g, yr, q, \omega) - t(g, yr, q, lb) \qquad \forall g \in \mathcal{G} \tag{9}$$
$$gDB^{+}(g, yr, q, \omega) \geq 0 \qquad \forall g \in \mathcal{G} \tag{10}$$

We constrain $gDisben^{+}$ using constraints (9) and (10) as part of determining each gentailer's risk adjusted profit.

Second Stage Objective Definition (yr, q, lb, ω)

$$gDB(g, yr, q, lb, \omega) =$$
$$\sum_{r \in \mathcal{R}} \Bigg[\text{d\_ret}(yr, q, lb, \omega, r) \cdot \text{d\_por}(g, r) \cdot (\text{p(r, yr, q, lb,} \omega) - \text{p\_ret}) \tag{11}$$
$$- \sum_{\substack{p \in \mathcal{P}, k \in \mathcal{K}, f \in \mathcal{F}:\\ \text{P\_G}(p)=g, \text{P\_R}(p)=r,\\ \text{P\_PT}(p)=k, \text{PT\_F}(k)=f}} \Bigg(p(r, yr, q, lb, \omega) \cdot y(p, yr, q, lb, \omega) \tag{12}$$
$$+ (\text{c\_marg}(p) + \text{c\_deliv}(p) + \text{c\_fuel}(f, yr)) \cdot y(p, yr, q, lb, \omega) \Bigg) \Bigg] \qquad \forall g \in \mathcal{G} \tag{13}$$

This equation determines the instantaneous negative profit each gentailer earns in each of the indexed times and scenario. Equation (11) refers to the revenue the gentailer g gains from selling electricity to their retail consumers and cost in purchasing this electricity on the spot market. Equation (12) gives the revenue gained from selling electricity on the spot market. Equation (13) refers to the cost of generation.

Second stage generation constraints (yr, q, lb, ω)

The maximum level of generation is depends by the level of investment in capacity expansion.

$$x(p) \cdot \left(1 - \text{d\_fof}(p, \omega) \sum_{\substack{k \in \mathcal{K}: \\ P\_PT(p)=k}} \text{d\_fom}(k, lb) \right) - y(p, yr, q, lb, \omega) \geq 0 \quad \forall p \in \mathcal{P} \quad (14)$$

$$y(p, yr, q, lb, \omega) \geq 0 \quad \forall p \in \mathcal{P} \quad (15)$$

Second Stage Industrial Objective(yr, q, lb, ω)

Here we describe the objective of industrial consumers of electricity which can purchase electricity from the spot market.

$$\min indDB(c, yr, q, lb, \omega) \qquad \forall c \in \mathcal{C} \qquad (16)$$

With this disbenefit defined by the negative of the potential short run loss the industrial consumer $c \in \mathcal{C}$ avoids by curtailing demand.

$$indDB(c, yr, q, lb, \omega) = \sum_{\substack{r \in \mathcal{R}: \\ I\_R(c)=r}} (p(r, yr, q, lb, \omega) - p\_ind(c))\delta\_ind(c, yr, q, lb, \omega) \qquad \forall c \in \mathcal{C} \qquad (17)$$

Second Stage Curtailment Constraints (yr, q, lb, ω)

$$0 \leq \delta\_ind(c, yr, q, lb, \omega) \leq \text{d\_ind}(c) \qquad \forall c \in \mathcal{C} \qquad (18)$$

The bounds on the curtailment of this type of demand are defined by the capacity of each generation plant.

Independent System Operator objective (yr, q, lb, ω)

The ISO sends electricity between nodes between nodes to maximize their own profit, transmitting electricity between these nodes optimally. The addition of the ISO ensures that given the available generation and consumption, that the welfare of the system is maximized.

$$\min isoDB(yr, q, lb, \omega) \qquad \forall yr \in \text{YEAR} \qquad (19)$$

The disbenefit in equation (19) is defined as the negative of the savings they make through their transmission decisions.

$$isoDB(yr, q, lb, \omega) = \sum_{r \in \mathcal{R}} p(r, yr, q, lb, \omega) \cdot \left[\left(\sum_{l \in \mathcal{L}} \text{n\_arc}(r, l)(f^{-}(l, yr, q, lb, \omega) - f^{+}(l, yr, q, lb, \omega)) \right) + loss(r, yr, q, lb, \omega) \right] \quad (20)$$

Second state transmission constraints (yr, q, lb, ω)

For the loss in equation (20), we assume quadratic losses giving (21).

$$loss(r, yr, q, lb, \omega) = \frac{1}{2}\sum_{l\in\mathcal{L}} |\text{n\_arc}(r,l)|\, \text{n\_loss}(l) \left(f^{+}(l, yr, q, lb, \omega) - f^{-}(l, yr, q, lb, \omega)\right)^{2} \quad \forall r \in \mathcal{R} \tag{21}$$

The network must follow Kirchhoff's voltage and current laws leading to the constraint which approximates this equation below. We must also fix one of the voltage angle variables at each time period and realization as we are concerned with the relative voltage angles.

$$(f^{+}(l, yr, q, lb, \omega) - f^{-}(l, yr, q, lb, \omega)) \cdot \text{n\_loss}(l) = \sum_{r\in\mathcal{R}} \text{n\_arc}(r,l)\theta(r, yr, qtr, lb, \omega) \quad \forall l \in \mathcal{L} \tag{22}$$

There is an upper bound on the power which can be safely transmitted across the lines. These bounds are defined below.

$$\text{n\_cap}(l) - \left(f^{+}(l, yr, q, lb, \omega) - f^{-}(l, yr, q, lb, \omega)\right) \geq 0 \quad \forall l \in \mathcal{L} \tag{23}$$

$$\text{n\_cap}(l) + \left(f^{+}(l, yr, q, lb, \omega) - f^{-}(l, yr, q, lb, \omega)\right) \geq 0 \quad \forall l \in \mathcal{L} \tag{24}$$

With the forward and backward part of the flow constrained to be positive:

$$f^{+}(l, yr, q, lb, \omega) \geq 0 \quad \forall l \in \mathcal{L} \tag{25}$$

$$f^{-}(l, yr, q, lb, \omega) \geq 0 \quad \forall l \in \mathcal{L} \tag{26}$$

Market MCP

Finally, the market equilibrium constraint which combine with the second stage objectives for each player to determine the spot market price at each node, ensuring that the spot market price of electricity is the marginal cost of the most expensive generation (or the marginal value of the highest valued demand not being met - which ever is higher). If there is insufficient generation supplied to a node, we curtail the remaining demand and set the the price to VOLL.

Second Stage - MCP Market(yr, q, lb, ω)

$$\begin{aligned}
0 \leq\ & \left[\sum_{\substack{p\in\mathcal{P}:\\ \text{P_R}(p)=r}} y(p, yr, q, lb, \omega)\right] + \delta(r, yr, q, lb, \omega) + \sum_{\substack{c\in\mathcal{C}:\\ \text{I_R}(c)=r}} \delta_ind(c, yr, q, lb, \omega) \\
+ & \left[\sum_{l\in\mathcal{L}} \text{n_arc}(r,l) \left(f^{+}(l, yr, q, lb, \omega) - f^{-}(l, yr, q, lb, \omega)\right)\right] - \text{d_ret}(yr, q, lb, \omega, r) \\
- & \left[\sum_{\substack{c\in\mathcal{C}:\\ \text{I_R}(c)=r}} \text{d_ind}(c)\right] - loss(r, yr, q, lb, \omega) \quad \perp \quad p(r, yr, q, lb, \omega) \geq 0 \quad \forall r \in \mathcal{R} \\
0 \leq\ & VOLL - p(r, yr, q, lb, \omega) \quad \perp \quad \delta(r, yr, q, lb, \omega) \geq 0 \quad \forall r \in \mathcal{R}
\end{aligned}$$

3 Results

This formulation was applied in the context of the New Zealand electricity market using the relevant data obtained from GEM as well as other estimated parameters. As the risk averse model is currently very difficult to solve using this formulation, we have limited its complexity. This is done by modeling 40 identical quarters, limiting the possible outcomes to four, and simplifying the network down to 2 nodes (figure 1) representing the north and south island (accumulating the demand and supply of the respective islands at these nodes).

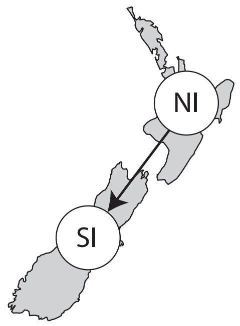

Figure 1: Representation of the New Zealand bulk electricity network

Along with the competitive risk averse model defined in section 2 other models were developed which were used to both verify the formulation and to later make comparisons. These models were as follows:

- Central plan with the objective of minimizing the overall expected welfare of the system
- Perfectly competitive model (defined in section 2)
- Perfectly competitive model with all electricity sold on the spot market (no retail)

With risk neutral and risk averse versions of each of these models solved and compared.

3.1 Solutions under each model

As we observe in figure 2, there is no difference in investment decisions under all risk neutral models. In fact this is always the case, as the central plan model directly corresponds to the perfectly competitive model and the vertical integration does not change which solution will maximise the expected profit (as its inclusion shifts the profit under each scenario independent of the actual decisions made).

However, when we include risk aversion into our model, we observe different investment decisions between each of the models described previously. This is because the risk averse models essentially scale the probability of each event from the perspective of each agent (with only one agent in the central plan) in different ways depending on which events lead to the worst outcomes (which are different under each of these models). This leads to the different investment decisions between each of the models shown in figure 3.

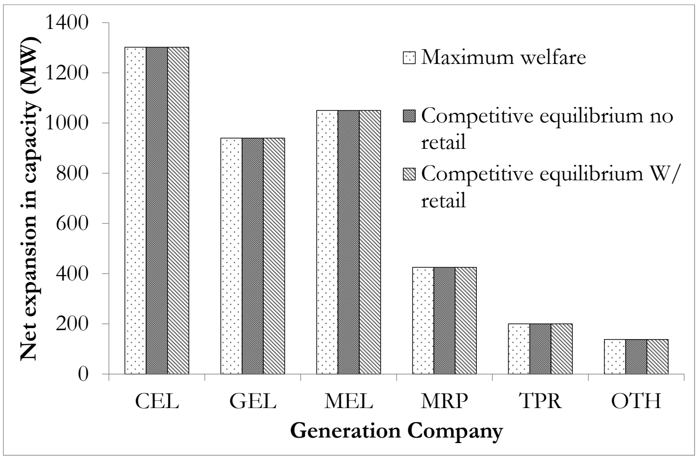

Figure 2: Investment decisions under the central plan the perfectly competitive model

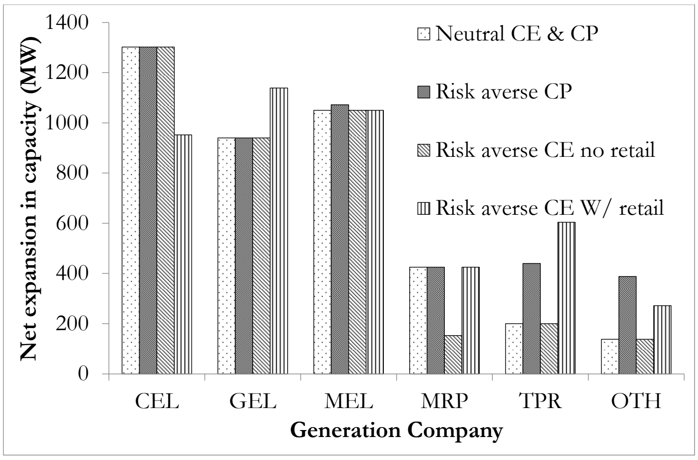

Figure 3: Expansion under each model with uncertainty of the resulting scenario

3.2 Welfare under each model

The overall cost to the system can be broken down into the following components.

- Investment in generation expansion;
- Operation and maintenance of generation;
- Generation of electricity;
- Curtailment of industrial demand;
- Curtailment of retail demand;

Figure 4 shows the breakdown of the expected overall cost under each model. The risk neutral models have the smallest expected cost to the system as their objectives each correspond to minimising this expected overall cost.

The risk averse central plan invests more in generation expansion as it places a larger weighting on times with insufficient supply. The risk averse competitive equilibrium without vertical integration invests less in generation as all agents now weigh they decisions more heavily on times of lower profits which occur during excess supply. The risk averse competitive equilibrium with vertical integration invests more in generation due to the overall trend of large losses when demand exceeds supply.

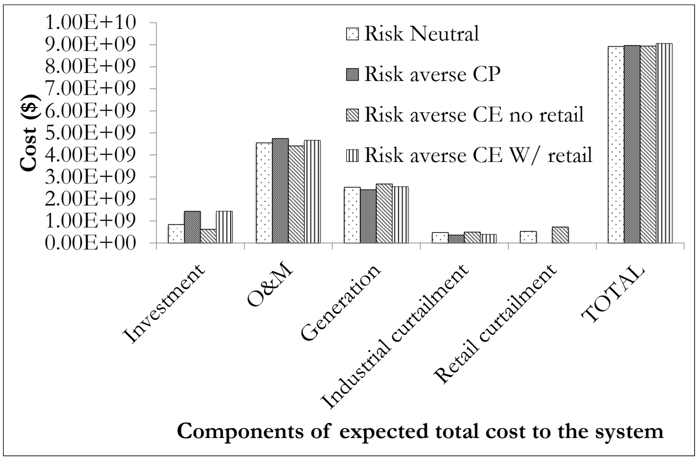

Figure 4: Breakdown of expected overall cost under each model

4 Conclusions

- A model has be formulated and implemented in GAMS which adapts the risk averse competitive model to generation expansion
- Results show that by using this risk measure:
  - We observe very different levels in expansion compared with a central plan using this risk measure (recalling that GEM is a central plan).
  - Each company's retail market share has a large impact on their profit under each scenario causing them to over-invest in our model.
  - Without retail share, companies tend to under invest with increased weighting on scenarios with a significant surplus of supply.

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