A Multi-commodity Flow Formulation for the Integrated Aircraft Routing, Crew Pairing, and Tail Assignment Problem

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Abstract

The integrated aircraft routing, crew pairing, and tail assignment problem consists of simultaneously finding a minimum cost set of aircraft routes and crew pairings such that each flight is covered by one aircraft and one crew.

A common problem when integrating airline planning stages is the long planning horizon of the crew pairing problem. We propose an approach in which crews initially are only told when they work. This enables us to generate an overall schedule much closer to the start of the planning horizon. Therefore, along with a short planning horizon, much more detailed and accurate overall schedules can be generated.

Due to the tail assignment aspect of the problem maintenance requirements have to be satisfied for each aircraft. Robustness of solutions is increased by using penalties on short connections.

We propose a mixed integer multi commodity flow formulation and report results for small instances.

Key words: Aircraft Routing, Crew Pairing, Tail Assignment, Muli-commodity Flow Formulation

1 Introduction

Airlines have been using operations research methods to tackle their complex planning and operational problems for decades. Due to the size of their operations not all planning decisions can be made simultaneously. In practice a sequential approach is used in the planning stage (Klabjan 2005). After a *flight schedule* is developed aircraft types are assigned to each flight leg in the *fleet assignment* problem. The goal is to match the capacity of the aircraft and the estimated number of passengers closely, thereby maximizing overall profit. Only few resources such as the number of available aircraft in each fleet are considered.

The following *aircraft routing* problem selects a minimal cost set of routes such that all flight legs are covered. Generic aircraft routes are generated by selecting a number of flights that have to be flown in sequence. While general maintenance requirements like days between maintenance checks have to be respected, the actual state of an aircraft, for example time spent flying so far, is ignored.

In the *crew pairing* problem generic pairings are generated such that the crew costs are minimized. A pairing is generated by selecting a number of flight legs that are serviced in succession by a crew while obeying several restrictions such as days off, limits on time spend working or layover requirements.

The pairings are put together to monthly rosters in the *crew rostering* problem. They are then assigned to actual crews based on personal preferences and needs. The crew rostering problem is solved approximately one month before the day of operations and only minor changes in the crew schedule are made after this step.

A few weeks before the day of operations the *tail assignment* problem is solved. Individual aircraft are assigned to routes generated in the aircraft routing problem. The planner needs to ensure that routes are feasible with respect to maintenance, taking into account the actual aircraft location and resource consumption at the beginning of the time horizon.

Because of the sequential nature it is very unlikely that the overall optimal or even a good solution can be obtained. Decisions in early planning stages like flight schedule generation or fleet assignment restrict the choices in later stages. For example in the fleet assignment problem the main issue is that no detailed information about aircraft and crews are available so the fleet assignment problem usually does not take into consideration that aircraft are temporarily unavailable due to maintenance checks. This may result in infeasibility in the aircraft routing problem, in which case flight legs have to be assigned to different aircraft types manually, increasing cost significantly. Also interdependences between stages, for example the issue of short connections in the aircraft routing and crew pairing problem, will result in higher cost when not considered in an integrated model.

The outline of this paper is as follows. In section 2 we give the motivation of our approach. Section 3 reviews integrated models that have been published. In section 4 we present a multi commodity flow formulation. We finish the paper with an outline of future work in section 5.

2 Motivation

A major disadvantage of the sequential approach is that it includes solving the aircraft routing and the crew pairing problem usually weeks or even months before the day of operations. Any decision making process is limited by the accuracy of the information available at the time of the decision. In the case of aircraft routing, information about the status and location of aircraft become significantly more accurate as the day of operations approaches. For this reason, it would be beneficial to delay any decisions as long as possible, while ensuring that subsequent planning stages can be carried out.

However, crews want to know their schedules some time in advance. Therefore the crew rostering problem is a major limiting factor to prolonging any decisions. It is usually solved approximately one month before the day of operations. This has two implications: first, the decisions of the aircraft routing problem and those of the crew pairing problem have to be made at least one month before the day of operations. Second, the minimum duration of a schedule is one month.

We propose an approach that can overcome this issue. Weeks before the day of operations an airline tells its crews on which days they have to work but not on which flights, i.e. only tells them the beginning and end of their pairings. Crew members then are able to manage their private lives while the airline can prolong aircraft routing and crew pairing decisions. Because crews are told when they have to work weeks before the final schedule is generated, it is the airline's responsibility to later generate pairings with durations close to the scheduled durations. If the airline fails to do so, the crew will have to get payed, despite not actually working on those days.

We develop the idea further by noting that there may be multiple crews stationed at the same base having the same start and end date. These crews can be considered identical in the planning process and are therefore represented by a *crew block*. Within a crew block the airline is free to make any assignments closer to the day of operations as long as the block start and end times are respected.

This approach impacts both issues raised above: the aircraft routing and the crew pairing problems do not need to be solved at least one month before the day of operations and the length of the schedule does not need to be one month. Instead, the length of the schedule now depends on negotiations between the airline and its employees. Crews still prefer knowing in advance on which days and which flights they have to work but may be more flexible if a higher financial reward is payed. We believe that the increased cost associated with having more flexible crews is more than offset by higher revenue resulting from more accurate operational schedules.

The duration of the schedule to be generated should not exceed 7 days as deviations from a longer schedule are to be expected. However, another 5 days worth of flying is included so that it can be guaranteed that crews are able to finish their pairings and that aircraft routes are feasible with respect to maintenance. Most published models solve a daily problem and then simply repeat the schedule for every day, neglecting the fact that the demand and schedule can be substantially different on weekends. Instead, we consider the actual flights scheduled in the next 12 days. The model is to be solved 4 days before the day of operations. At that time it should be possible to project location and status of crews and aircraft accurately. On the other hand sufficient time will be available to evaluate the solution and possibly resolve the problem.

Solving an integrated model this close to the day of operations facilitates considering individual aircraft, i.e. tail numbers. This addresses another major disadvantage of models that are solved a long time before the day of operations. In recent integrated models (see section 3) the aircraft routing problem ensures that maintenance feasible routes are generated. However, these routes are generic and do not consider the current state of an aircraft on the day of operations. Often the schedule has to be changed because aircraft are at a different location than expected at the time of the planning process or they require maintenance earlier than anticipated. Four days before the day of operations information will be available that allows accurate prediction of state and location of an aircraft. Therefore, instead of generating generic routes we propose to generate routes for each individual tail number. These routes can represent specific maintenance requirements, eliminating the need to assign aircraft and possibly having to change the schedule because a feasible assignment is not possible.

In summary, we integrate aircraft routing, tail assignment and crew pairing generation. The model is to be solved daily with a rolling horizon, enabling us to consider current developments in resource consumption or deviations from earlier schedules.

3 Literature review

In scholarly publications a number of integrated models have been developed in the past. (Desaulniers et al.) and (Barnhart et al. 1998) semi-integrate fleet assignment and aircraft routing. In both cases the models are solved using the branch-and-price method. In the former no maintenance is considered, therefore feasibility can not be guaranteed. Flights are allowed to depart within a certain time window, resulting in lower cost compared to fixed departure times. The latter considers strings of flights with maintenance opportunities attached to the end. The cost of assigning an aircraft type to a flight leg is considered.

(Klabjan et al.) semi-integrate aircraft routing and crew pairing. They reverse the order by first solving the crew pairing problem instead of the aircraft routing problem. Plane-count constraints are introduced to the crew pairing problem, ensuring feasibility of the aircraft routing problem under the assumption that maintenance is performed over night when all aircraft are on the ground. In addition, flight departure times are allowed to vary within time windows.

A model that fully integrates aircraft routing and crew pairing is developed by (Cordeau et al. 2001). The aforementioned issue of short connections is considered here for the first time. The resulting large number of constraints are handled by Benders decomposition, with aircraft routing as the master problem. The solution process iterates between the master problem and a subproblem that solves the crew pairing problem. Both the Benders master and subproblem are solved by Column Generation. No cost are associated with aircraft routes, reducing the aircraft routing problem to a feasibility problem. This neglects the fact that with more frequent maintenance aircraft are unavailable more often and higher maintenance costs are incurred.

(Cohn and Barnhart 2003) also present a model for the integrated aircraft routing and crew pairing problem. As in (Cordeau et al. 2001) aircraft routing cost are neglected and the aircraft routing reduces to a feasibility problem. However, they develop this idea further by realizing that routes that differ only by the order of the legs flown can be represented by a single column in the so called extended crew pairing problem. This reduces the number of variables in the model significantly but comes at the cost of having to solve multiple aircraft routing problems that generate these columns. However, the approach is inapt if maintenance cost need to be considered because in this case the routes differ not only by the order of legs flown. The extended crew pairing problem is solved by a branch-and-price algorithm. The crew pairing and aircraft routing solutions are generated in pricing problems.

The model of (Cordeau et al. 2001) was developed further by (Mercier, Cordeau, and Soumis 2005) who increased solution robustness by penalizing connections that are likely to cause delays if they are not performed by the same aircraft and crew. The idea is a generalization of the concept used for short connections. The authors also show that solving the aircraft routing problem as the Benders master problem and the crew pairing problem as the subproblem is beneficial because fewer Benders cuts need to be generated. This can be attributed to the fact that with an aircraft routing subproblem mostly feasibility and only little optimality information is transferred to the master problem.

The latter model is further enhanced by (Mercier and Soumis) by introducing time windows to the formulation. A solution algorithm combining Benders decomposition, column generation and a dynamic constraint generation procedure is developed and proved to be very efficient for their problem.

(Sandhu and Klabjan 2007) integrate fleet assignment and crew pairing while considering some aircraft routing aspects. The model ensures that a pairing is assigned to a single fleet type only, thus permitting different sizes and qualifications of crews. Only plane-count constraints are included in the formulation which means that maintenance feasibility can not be guaranteed. The model is solved by Benders decomposition as well as a combination of Lagrangian relaxation and branch-and-price. The first method finds good solutions quickly but is outperformed by the latter if more solution time is available.

(Weide 2009) uses an iterative procedure in which the aircraft routing and the crew pairing problem are solved alternately. Short and restricted connection rules are considered but because of the iterative nature not all of these connections are included. A daily flight schedule is generated and maintenance is assumed to be carried out over night when aircraft are on the ground.

(Papadakos 2009) presents a model that is based on (Cordeau et al. 2001) but fully integrates fleet assignment, aircraft routing, and crew pairing. Similar to (Sandhu and Klabjan 2007) crew pairings are fleet dependent. The model is solved using a combination of Benders decomposition and accelerated column generation.

The literature suggests column generation or Bender's decomposition or even a combination of both as suitable algorithms to solve integrated airline problems. However due to the advances in general mixed integer programming in the last decade we decided to explore if such standard methods have become suitable as well.

4 Mathematical model

The model is based on a muli-commodity flow formulation where each crew block and each aircraft are represented by one commodity. The nodes in the network represent flight legs, while the arcs represent connections between flight legs. Additionally, one source and one sink node is added for each commodity.

Sets

B	The set of all crew blocks
R	The set of all individual aircraft
Ω	The set of all types of maintenance checks
Δ	The set of all resources under consideration
Δ_ω	The set of resource which apply to the check type $\omega \in \Omega$
N	The set of all nodes
N_{Legs}	The set of nodes which represent actual flight legs
N_{AC}^+	The set of aircraft source nodes r^+
N_{AC}^-	The set of aircraft sink nodes r^-

C_{Crew}	The set of all crew connections
$C_{\text{Crew}}^{\text{Duty}}$	The set of duty connections for crews
$C_{\text{Crew}}^{\text{Restr}}$	The set of restricted connections for crews
$C_{\text{Crew}}^{\text{Lay}}$	The set of layover connections for crews
C_{AC}	The set of aircraft connections

Parameters

$\text{Block}(i)$	The block hours for leg $i \in N_{\text{Legs}}$
$\text{Wait}(i,j)$	The total crew waiting time at an airport associated with connection $(i,j) \in C_{\text{Crew}}$ including briefing time, waiting and transit time, but not including debriefing time
MaxDuty	The maximum allowable duty hours in a duty
$\text{Penalty}(i,j)$	A penalty for assigning a crew block to a restricted connection $(i,j) \in C_{\text{Crew}}^{\text{Restr}}$ while not assigning an aircraft to the same connection
$\text{MaintCost}_{ij}^{\omega}$	Cost of carrying out a maintenance check of type $\omega \in \Omega$ on connection $(i,j) \in C_{\text{Craft}}$
$\text{LayCost}(i,j)$	The cost of a layover on connection $(i,j) \in C_{\text{Crew}}^{\text{Lay}}$, which includes transport, hotel, and meal allowance cost.
$\text{Credit}(i)$	The applicable credit for leg $i \in N_{\text{Legs}}$
MinCredit	The minimum amount of credit for a duty
CreditCost	The cost of one unit of credit for one crew
DeBriefTime	The duration of a debriefing at the end of a duty
$\text{Used}_{\text{turn}}^{\delta}(i,j)$	The amount of resource $\delta \in \Delta$ used on connection $(i,j) \in C_{\text{Craft}}$
$\text{Used}_{\text{leg}}^{\delta}(i)$	The amount of resource $\delta \in \Delta$ used on leg $i \in N_{\text{Legs}}$
$\text{StartVal}^{\omega\delta}(r)$	The amount of resource $\delta \in \Delta_{\omega}$ used since the last maintenance of type $\omega \in \Omega$ for aircraft $r \in R$ at the start of the planning period
$\text{MaxMaint}^{\omega\delta}$	The maximum amount of resource $\delta \in \Delta_{\omega}$ which can be accumulated before carrying out a maintenance check of type $\omega \in \Omega$

Decision variables

$$y_{ijb} = \begin{cases} 1 & \text{if crew block } b \in B \text{ uses connection } (i,j) \in C_{\text{Crew}} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijr} = \begin{cases} 1 & \text{if aircraft } r \in R \text{ uses connection } (i,j) \in C_{\text{Craft}} \\ 0 & \text{otherwise} \end{cases}$$

$$w_{ij} = \begin{cases} 1 & \text{if both plane and crew use connection } (i,j) \in C_{\text{Crew}}^{\text{Restr}} \cap C_{Craft} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ij}^{\omega} = \begin{cases} 1 & \text{if maintenance } \omega \in \Omega \text{ is scheduled on connection } (i,j) \in C_{\text{AC}} \\ 0 & \text{otherwise} \end{cases}$$

d_i The number of duty hours accumulated within a duty by a crew block at the end of leg $i \in N_{\text{Legs}}$

a_i The number of credit hours accumulated within a duty by a crew block at the end of leg $i \in N_{\text{Legs}}$

$\overline{a_{ij}}$ The number of credit hours accumulated by the end of a duty for a crew block that finishes its duty at the end of leg i and then connects to j, $\forall (i,j) \in C_{\text{Crew}}^{\text{Lay}}$ or $j \in N_{\text{AC}}^-$

$m_i^{\omega\delta}$ The amount of resource $\delta \in \Delta$ accumulated since last maintenance of type $\omega \in \Omega$ at the end of leg $i \in N_{\text{Legs}} \cup N_{\text{AC}}^+$

4.1 Objective function

The objective is to minimize the cost associated with a flight schedule and to increase the robustness of the solution:

$$\begin{aligned}\text{Minimize} \quad & \sum_{\substack{\omega\in\Omega,\\(i,j)\in C_{\text{AC}}}} \text{MaintCost}_{ij}^{\omega} * z_{ij}^{\omega} \\ & + \sum_{(i,j)\in C_{\text{Crew}}^{\text{Lay}}} \text{LayCost}(ij) * \sum_{b\in B} y_{ijb} \\ & + \sum_{(i,j)\in C_{\text{Crew}}} \text{CreditCost} * \overline{a}_{ij} \\ & + \sum_{(i,j)\in C_{\text{Crew}}^{\text{Restr}}} \text{Penalty}(i,j) * w_{ij}\end{aligned} \tag{1}$$

The first term represents the maintenance cost if maintenance is carried out on a connection. The second term counts the layover cost, while the third captures the cost incurred by scheduling a certain duty. The last term is incurred every time an aircraft and a crew do not stay together on a restricted connection. All of these costs and penalties are only applied to flight legs and connections that occur in the first seven days of the planning horizon.

4.2 Constraints

We require that every flight has to be covered by exactly one aircraft and one crew block. This is achieved by using standard set partitioning constraints. Additionally we need flow conversation constraints and constraints that ensure that each commodity leaves its source. Another set of constrains deals with the issue of short connections, see (Cordeau et al. 2001). They require that if a crew uses a short connection an aircraft has to use the same connection as well. In other words, the same crew and aircraft are assigned to consecutive flights, i.e. no aircraft swap occurs.

To increase robustness a penalty term is incurred every time a crew or an aircraft uses a restricted connection without the other one doing so as well. This increases robustness as fewer aircraft changes occur. Restricted connections have a duration of 45 to 90 minutes.

$$\sum_{r \in R} x_{ijr} \leq w_{ij}, \qquad \forall\ (i,j) \in C^{\text{Restr}}_{\text{Crew}} \cap C_{\text{AC}} \tag{2}$$

$$\sum_{b \in B} y_{ijb} \leq w_{ij}, \qquad \forall\ (i,j) \in C^{\text{Restr}}_{\text{Crew}} \cap C_{\text{AC}} \tag{3}$$

4.2.1 Constraints relating to the length of a crew pairing

Regulations in Australia require that a crew only works up to 8 *block hours* and up to 11 *duty hours* per duty. Block hours are defined as time spent flying plus taxi time, while duty hours are the block hours plus the time spent at an airport between flights including briefing time. Duty hour restrictions are modeled using constraints:

$$d_i \leq \text{MaxDuty} - \text{DeBriefTime}, \qquad \forall i \in N_{\text{Legs}} \tag{4}$$

$$d_i \geq \text{Block}(i) + \sum_{\substack{b \in B,\, j \in N| \\ (j,i) \in C_{\text{Crew}}}} \text{Wait}(j,i) * y_{jib}, \qquad \forall i \in N_{\text{Legs}} \tag{5}$$

$$d_j - d_i \geq \text{Block}(j) + \text{Wait}(i,j) - M(1 - \sum_{b \in B} y_{ijb}), \qquad \forall (i,j) \in C^{\text{Duty}}_{\text{Crew}} \tag{6}$$

The first constraint limits the number of duty hours, while the second ensures that the book-keeping variable starts with the correct start value. The last constraint increments the variable along an arc if a crew uses the corresponding connection. Valid big M values are $M = \text{MaxDuty} - \text{DeBriefTime}$. Block hour requirements are modeled in a similar way, however they don't include a Wait term in the constraints.

Other regulations limit the number of block hours to 30 in a 7 day period for every crew. Because we are not considering detailed crew scheduling, we approximate this rule by applying the limit to every pairing, which have a maximal duration of 5 days. This rule is implemented in a similar fashion as the duty hour restrictions, constraints (4) - (6). The only difference is that the total block hours value increments along all crew connections including layovers and that no Wait term exists.

4.2.2 Constraints relating to crew cost

Applicable credit is a concept used to describe the amount of pay that a crew member receives. A *credit* value is associeated with every flight leg depending on its duration. The actual crew payments are then based on a fixed salary, with overtime paid if the number of credit hours exceeds a given value. For the time frame considered here, an approximation is used where applicable credit is charged at a fixed rate per hour per crew. Similar constraints as above are required:

$$a_j - a_i \geq \text{Credit}(j) - M(1 - \sum_{b \in B} y_{ijb}), \qquad \forall (i,j) \in C^{\text{Duty}}_{\text{Crew}} \tag{7}$$

$$a_i \geq \text{Credit}(i), \qquad \forall i \in N_{\text{Legs}} \tag{8}$$

The credit values at the end of each duty are required for costing purposes. Constraint (9) assigns the accumulated credit to variables representing the end of duties.

$$\overline{a_{ij}} \geq a_i - M(1 - \sum_{b \in B} y_{ijb}), \qquad \forall (i,j) \in C^{\text{Lay}}_{\text{Crew}} \text{ or } j \in N^{-}_{\text{AC}} \tag{9}$$

If an airline were to schedule too many short duties or pairings, it could become infeasible to schedule all flights including leave, training and days off. One method currently employed by airlines to ensure a sensible set of trips is to apply a minimum amount of applicable credit to each duty. Each duty will incur cost equal to this value or its true amount of credit, whichever is greater, thus penalizing the creation of too many duties. The following constraint is imposed on every arc that represents the end of a duty.

$$\overline{a_{ij}} \geq \text{MinCredit} * \sum_{b \in B} y_{ijb}, \qquad \forall (i,j) \in C_{\text{Crew}}^{\text{Lay}} \text{ or } j \in N_{\text{AC}}^{-} \tag{10}$$

4.2.3 Constraints relating to maintenance decisions

Maintenance requirements are modeled in a similar way as well. The book keeping variables are initialized with the correct values for each aircraft at the beginning of the time horizon

$$m_{r^+}^{\omega\delta} = \text{StartVal}^{\omega\delta}(r), \qquad \forall r \in R,\ \omega \in \Omega,\ \delta \in \Delta_\omega \tag{11}$$

The maintenance variables have upper limits.

$$m_i^{\omega\delta} \leq \text{MaxMaint}^{\omega\delta}, \qquad \forall i \in N_{\text{Legs}},\ \omega \in \Omega,\ \delta \in \Delta_\omega \tag{12}$$

Variables are incremented along any arc that an aircraft travels. However, the variables are not incremented if maintenance occurs at the same time, i.e. $z_{ij} = 1$. A sufficient big-M value is $M = \text{MaxMaint}^{\omega\delta}$.

$$m_j^{\omega\delta} - m_i^{\omega\delta} \geq \text{Used}_{\text{turn}}^{\delta}(i,j) + \text{Used}_{\text{leg}}^{\delta}(j) - M(1 - \sum_{r \in R} x_{ijr} + z_{ij}^{\omega}), \tag{13}$$

$$\forall (i,j) \in C_{\text{AC}},\ \omega \in \Omega,\ \delta \in \Delta_\omega$$

Scheduling maintenance resets the book-keeping variables to the value of the current connection.

$$m_j^{\omega\delta} \geq \text{Used}_{\text{turn}}^{\delta}(i,j) + \text{Used}_{\text{leg}}^{\delta}(j) - M(1 - z_{ij}^{\omega}), \tag{14}$$

$$\forall (i,j) \in C_{\text{AC}},\ \omega \in \Omega,\ \delta \in \Delta_\omega$$

Maintenance can only be carried out on connections that have a sufficient duration and that occur at a station that has appropriate equipment. No maintenance must be scheduled on a connection which is not used by an aircraft as well.

$$\sum_{\omega \in \Omega} z_{ij}^{\omega} \leq \sum_{r \in R} x_{ijr}, \qquad \forall (i,j) \in C_{\text{AC}} \tag{15}$$

The constraint also restricts the number of maintenance checks per connection to one. This aims at distributing the maintenance checks so that not too many are scheduled at one station at a time.

5 Results and future work

Results are pending and will be presented at the conference. However initial experiments have shown that constraints (9) and (10) make the problem harder to solve, possibly destroying the multi-commodity flow problem structure. Therefore if the results will be discouraging excluding constraints (7) through (10) may be considered.

The authors expect the problem to be too hard to be solve in reasonable time for medium and large instances, i.e. up to 2000 flight, 30 aircraft, and 100 crew blocks. Therefore other algorithms have to be explored. An obvious choice would be methods that were proposed in the literature (see Section 3), namely column generation and Bender's decomposition.

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