

Optimal Paths in Real Multimodal Transportation Networks: An Appraisal Using GIS Data from New Zealand and Europe

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Abstract

The most convenient route to connect two locations is often a mix of different transportation systems. For instance, a user can make intercity trips by selecting a combination of transport modes such as car, rail, ship or airplane. In this case the transportation system is said to be multimodal. In this paper, real multimodal transportation systems are experimentally analyzed. Real road–rail networks from Denmark, Hungary, Spain, Norway and New Zealand are built based on a set of digitized transportation maps obtained from several GIS libraries. These networks are modelled as coloured–edge graphs to be used as main input by a multimodal Dijkstra’s algorithm that computes a set of optimal paths. The cardinality of the resulting set is at the core of the approach tractability. It is concluded that vertices connectivity and network shape considerably affect the total number of optimal paths.

Key words: Multimodal network, coloured–edge graph, GIS libraries, transportation networks.

1 Introduction

A multimodal network (MMN) is a transportation system that considers two or more transport choices for connecting locations (vertices) in a network. Freight and urban transportation are two application fields in which multimodal networks are extensively found. In (Bontekoning, Macharis, and Trip 2004) freight transportation is thoroughly reviewed for identifying possible research trends. The authors here conclude research in multimodal freight transportation is still in a pre-paradigmatic phase. Urban transportation deals with the movement of passengers in urban areas considering variables such as congestion levels (flow), public fares, transport modes (bus, subway, private car or bicycle), service demand and user behaviour. Some

overviews about urban transportation are found in (Boyce 2007), (Lee 1994), (Wenger 1994) and (Nagurney 1984). Despite their popularity as multimodal research field, freight and urban transportation are not the only areas where multimodal networks can be used as a modelling tool. Application studies into computer networks (Nigay and Coutaz 1993), biomedicine (Heath and Sioson 2007) and manufacturing (Medeiros et al. 2000) can be noticed in the literature too.

One problem associated to MMN is the determination of a shortest path from an origin to a destination. The shortest path problem has been extensively studied in both practice and theory. Likewise, a large number of algorithms have been developed by different fields of enquire such as operation research and computing sciences. When dealing with algorithms for multimodal networks, researchers and practitioners typically opt for techniques based on variations of classical approaches such as Dijkstra’s algorithm or Bellman–Ford method (a description of these approaches is found in (Lawler 2001)). However, a direct application of these approaches generates a shortest path that does not consider the multimodal traits of the network as part of the analysis. Thereby, an optimal combination of means of transport is an outcome of the shortest path itself. Consequently, a multimodal network cannot be treated as a “unimodal” because each edge includes an additional variable (the mode) that has to be included as part of the analysis.

An appraisal is carried out in this paper to analyze the comportment of the shortest path problem in real MMNs. The employed modelling approach takes a coloured–edge graph that represents modes, cities and intercity links by colours, vertices and edges. This graph is used by a generalization of Dijkstra’s algorithm that computes a set of optimal paths. This set of paths is the key of the model’s tractability so that experiments are set for tracing its value. Unlike others approaches, the presented modelling and computational techniques are able to keep the multimodal traits of the network throughout the analysis. The results indicate that the total number of optimal paths is significantly influenced by the shape and connectivity of the network.

The remainder of this paper is organized in four further sections. First the modelling approach and the algorithm are described in section 2. The experimental setup is described in section 3. Results are given in section 4. Finally, section 5 provides conclusions and future work in this research field.

2 Model and Algorithm

2.1 The Coloured–Edge Graph Model

The coloured–edge graph is a modelling tool introduced by (Enzor and Lillo 2009) for the modelling of MMNs. This graph modelling approach labels edges with colours for representing a specific attribute such as a transport mode. In a real transportation system, two locations might be connected by several modes. For this case, the coloured–edge graph allows the use of multiple edges. These edges can be directed or undirected as well as weighted or unweighted. In this research weighted coloured–edge digraphs are utilized to model real multimodal transportation networks from several countries.

In their work (Enzor and Lillo 2009) formally define a weighted coloured–edge graph as $G = (V, E, \omega, \lambda)$ which is a directed graph (V, E) with vertex set V and edge set E , a weight function $\omega: E \rightarrow \mathbb{R}^+$ on edges, and a colour function $\lambda: E \rightarrow M$

on edges. M is a finite set of colours with $k = |M|$. Each edge $e_{uv} \in E$ joining vertices u and v has a positive weight $\omega(e_{uv})$ and a colour $\lambda(e_{uv})$. For any colour $i \in M$ and for any path p_{uv} between two vertices u and v , the path weight $\omega_i(p_{uv})$ in colour i is defined as $\omega_i(p_{uv}) = \sum_{e_{uv} \in p, \lambda(e_{uv})=i} \omega(e_{uv})$. The total path weight is represented as a k -tuple $(\omega_1(e_{uv}), \dots, \omega_i(e_{uv}), \dots, \omega_k(e_{uv}))$, giving the total weight of the path in each colour.

Only limited research has been conducted in the area of coloured-edge graphs. (Climaco, Captivo, and Pascoal 2010) studied the number of spanning trees in a weighted graph whose edges are labelled with a colour. This work defines weight and colour as two conflicting criteria. Hence, the proposed algorithm generates a set of nondominated spanning trees. The computation of coloured paths in a weighted coloured-edge graph is investigated by (Xu et al. 2009). The main feature of their approach is a graph reduction technique based on a priority rule. This rule basically transforms a weighted coloured-edge multidigraph into a coloured-vertex digraph by applying algebraic operations upon the adjacent matrix. Additionally, (Xu et al. 2009) provide an algorithm to identify coloured source-destination paths. Nevertheless, the algorithm is not intended for general instances because it just works with the number of edges as a path weight. Furthermore, only paths having consecutive edges with distinct colours are considered.

2.2 Multimodal Dijkstra's Algorithm

A weighted coloured-edge graph can produce a factorial number of source-destination paths. For instance, the total number of paths in a complete coloured-edge graph is $O(k^{n-1}(n-2)!)$. This can be proved by applying a basic counting argument. However, the question is how many of these paths are optimal (shortest). At first glance, traditional shortest path algorithms might be able to provide an answer. Nevertheless, these procedures were originally designed for "unimodal" networks. Thus, a direct application of such algorithms on a coloured-edge graph produces outcomes that do not take transport modes into consideration. This section explain a procedure designed by (Ensor and Lillo 2009) that extracts a set of optimal path from a weighted coloured-edge graph base on a general version of the well-known Dijkstra's algorithm. Firstly, some notation need to be introduced.

Let \mathcal{M}_{uv} be the set of shortest paths joining two vertices u and v in a weighted coloured-edge graph. This set can be extracted from a weighted coloured-edge graph by defining a partial order relation on weight tuples. As a result, a smaller set of paths is obtained. Each k -tuple in \mathcal{M}_{uv} is Pareto efficient which means that a weight in a tuple cannot be improved (or worsened) without worsening (or improving) the same weight of another. The cardinality of \mathcal{M}_{uv} is a key factor in determining the tractability of the model.

The computation of \mathcal{M}_{uv} is performed in (Ensor and Lillo 2009) by a generalization of Dijkstra's algorithm. Unlike its classic counterpart, the multimodal Dijkstra's algorithm (MDA) has a partially ordered data structure to manage path weights. The algorithm takes as input a coloured-edge network G and a source vertex s . It commences at s with the empty path and relaxes each edge that is incident from the source vertex s , adding the single edge paths to the queue. At the front of the queue will be a shortest path estimate to some vertex v adjacent to s . Next, the algorithm determines the shortest paths to v (note that there are more than one) and relaxes edges incidents to v . This iterative process finishes when the queue has

no more paths to be compared.

3 Experimental Setup

The experimental study collected vector data information about Denmark, Hungary, Spain, Norway and New Zealand from a GIS library (Geofabrik 2010). These countries were selected based on their similarities in shape and number of locations. For example, New Zealand has resemblances with Norway in shape and number of vertices. Both countries have a long shape and a number of locations between 100 and 200. In addition, one interest of this paper was to establish the extent in which a european multimodal transportation system differs from the New Zealand one.

The multimodal networks were stored and maintained as a set of vertices and bidirectional links. A network dataset for each mode was generated by firstly snapping vertices (towns and cities) to network features according to a tolerance radius. Secondly, a connectivity map was created by an ad-hoc algorithm that iterates itself through vertices. As a spin-off, this algorithm also calculated the real intercity distances as decimal geographic degrees. Additionally, airways were added as a third mode for Norway and New Zealand. Straight distances between airports were used as edge length in this case so that airports had to be snapped to cities to build a connectivity map. Airway data was obtained from (OpenFlights 2010). Characteristics of the resulting networks are shown in Table 1.

Table 1: Characteristics of the Networks

Network	Country	Vertices	Edges	Modes
1	Denmark	124	1284	Road,Rail
2	Hungary	305	7418	Road,Rail
3	Spain	901	5326	Road,Rail
4	Norway	122	641	Road,Rail,Airways
5	New Zealand	183	1436	Road,Rail,Airways

As an illustration, Figure 1 displays the Hungary roadway system which is composed by motorways and primary roads. Likewise, Figure 2 yields a view of New Zealand airway system.

The reported runtimes corresponds to CPU times by computing the total number of shortest path trees (\mathcal{M}_{uv} cardinality) from a source vertex. Each of these trees can be composed by any number of transport modes. Networks were all tested on a standard double core desktop computer of 1.86 GHz and 1.99 GB of RAM.

The calculation of \mathcal{M}_{uv} cardinality was settled upon two different source vertex scenarios. Scenario 1 considered the capital city of each country as source vertex whereas Scenario 2 uses an extreme city as source. For instance, Wellington and Invercargill were picked as source vertices for scenarios 1 and 2 respectively in the New Zealand's case. Besides, the algorithm was responsible by reporting the total number of processing paths (total number paths taken by the iterative subroutine of the MDA) as well as average and maximum \mathcal{M}_{uv} cardinality. Average cardinality is calculated by averaging all vertices' cardinalities whereas the maximum cardinality corresponds to the largest value of \mathcal{M}_{uv} cardinality among vertices.



Figure 1: Hungary roadway system.

4 Results

Results for Scenario 1 are shown in Table 2. CPU times are given in seconds.

One fact that Table 2 evidences is that just a small number of paths become optimal in comparison to the number of processing paths taken by the algorithm. No more than 2% of such paths became optimal for the studied multimodal networks. This is a promising result from a tractability viewpoint.

Spain and New Zealand obtained the largest values of \mathcal{M}_{uv} cardinality. What these countries have in common is a high level of network overlap between road and rail as well as a high number of cities located along such overlaps. These features

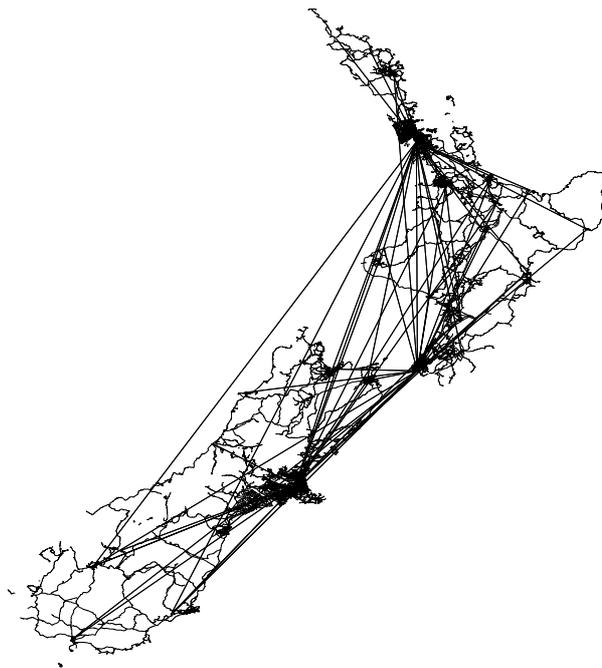


Figure 2: New Zealand airway system.

Table 2: Results for multimodal networks from vector data: Scenario 1

Network	Source city	Average \mathcal{M}_{uv} cardinality	Maximum \mathcal{M}_{uv} cardinality	Processing Paths	CPU time
1	Copenhagen	56	171	10704	0.515
2	Budapest	69	227	53587	6.024
3	Madrid	133	1039	181976	32.216
4	Oslo	41	147	6248	0.158
5	Wellington	759	9342	611230	311.184

together induce a high number of optimal paths because some network sections resemble a coloured-edge chain. Coloured-edge chains are important subgraphs in a general coloured-edge graph because they are able to elicit a worst case scenario. In their work (Ensor and Lillo 2010) show the worst case of a weighted coloured-edge graph occurs when a hamiltonian coloured-edge path (or coloured-edge chain) concentrates most of the lowest weights. In addition, weights in this chain have to satisfy an special equality condition. When these two requirements come together, the total number of optimal paths is $O(k^{n-1})$. In practical terms, those cities (or towns) that require a greater number of intermediate connections to be reached are prone to generate an elevated number of shortest coloured-edge paths. To envision the concept of network overlap, Figure 3 shows road and rail networks for New Zealand.

On the other hand, the rich variety of network links presented in Denmark, Hungary and Norway reduce overlap so that the number of optimal paths tend to be lower. Moreover, maximal cardinalities were found in remote cities (or towns) with no direct link from the sources. For example, Frederikshavn and Rakamaz were the locations reporting the maximum number of optimal paths for Denmark and Hungary, respectively. This indicates the number of optimal paths is affected

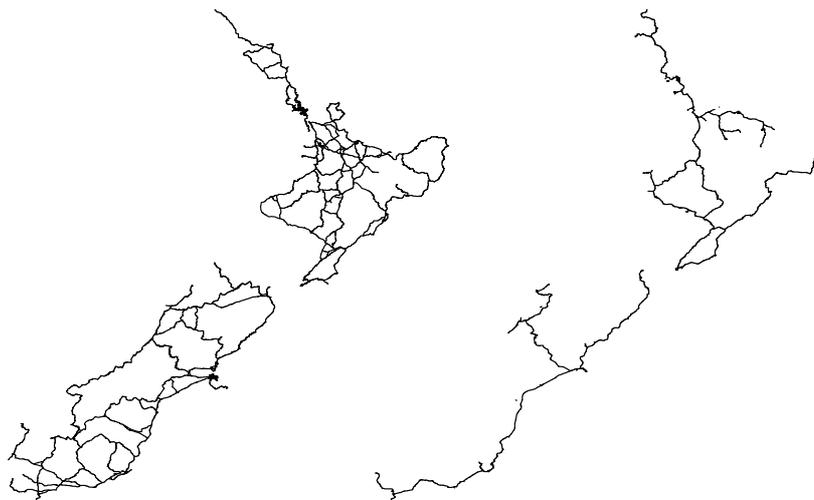


Figure 3: New Zealand road (left) and rail (right) networks. Note the high level of overlap in the east side of the country.

whether the location of the source is changed. A special case is a vertex located at one of the country extremes.

Scenario 2 was set to analyze the impact that source location has on the cardinality of \mathcal{M}_{uv} . Table 3 summarizes the corresponding results. The selected location were extreme points situated at one of the four cardinal points. Spain and New Zealand again concentrates the highest numbers of paths. Here, maximum cardinalities were obtained in San Fernando (Spain) and Riverton (New Zealand). Observe that the number of transport modes is not changing the cardinality pattern. Although more modes do increase \mathcal{M}_{uv} cardinality, the overlap is maintained since the configuration of the networks remains unchanged in each mode.

Table 3: Results for multimodal networks from vector data: Scenario 2

Network	Source city	Average \mathcal{M}_{uv} cardinality	Maximum \mathcal{M}_{uv} cardinality	Processing Paths	CPU time
1	Hanstholm	76	332	20009	1.262
2	Csenger	149	600	135218	68.423
3	Tarroella	423	1864	606609	507.657
4	Ergersund	78	245	11674	0.332
5	Kaitaia	5969	33246	1644768	20572.52

5 Conclusions

Real-world multimodal networks were computationally investigated in this work. The experiments were based on digitized road, rail and airways maps from Denmark, Hungary, Spain, Norway and New Zealand. Each map was pre-processed to be used as input by a multimodal version of Dijkstra’s algorithm that produces a set of optimal paths (\mathcal{M}_{uv}). The cardinality of this set was the main variable to be analyzed because of its influence on the model’s tractability. Such cardinality turn out to be higher and significantly concentrated on cities situated far away from the sources in those countries whose transportation systems exhibited a greater level of overlap. Overlap produces that certain sections of a multimodal network resemble a coloured-edge chain. These chains were proved by (Ensor and Lillo 2010) to be the cause of an exponential number of optimal paths in a coloured-edge graph.

Computational times were reasonable considering that the multimodal Dijkstra’s algorithm was implemented with a basic data structure (priority queue). Multimodal networks in Scenario 2 required longer runs than Scenario 1 due to the relocation of the sources. This relocation effect indicates higher number of optimal path could be needed to reach distant cities (or towns) when the source vertex is located at the very extreme of a country. Scenario 2 is besides attesting that country shapes are able to alter the number of optimal paths. Longer and slimmer shapes are thus closer to behave as a coloured-edge chain. Hereby, the special shape of New Zealand is also accounting for the elevated number of optimal path found for its extreme locations. Although Norway has a shape resemblance with New Zealand, the lower number of optimal paths is explaining by different circumstances: (1) Norway’s rail system is not able to connect the entire country. Rail roughly covers just 20% of the territory. (2) The multimodal transportation system of Norway goes from dense to very sparse

as a user moves from the south to the north. Road is predominantly defining the connectivity in the north. (3) The number of airways is much lower than in New Zealand. New Zealand has about 116 different air connections whereas Norway has just 62.

There were not remarkable differences in using two or three transport modes. This suggests that shape and connectivity are more determining factors in the tractability of these networks rather than the value of k .

Future work in this field can tackle the analysis of \mathcal{M}_{uv} cardinality on larger multimodal transportation datasets. This is particularly useful in assessing the correlation between country shape and tractability. Another issue is related to the distribution of weights. For instance, a worst case can be built by taking a coloured-edge chain with pure Euclidean distances as edge weights. However, real distances are far to be pure Euclidean in real transportation systems. Thereby, real distances could be causing a reduction in the number of processing paths.

Improvements on the algorithm are required to speed calculations up. Therefore, faster computational techniques such as parallel computing and ad-doc data structures could result in much faster implementations of the multimodal Dijkstra's algorithm.

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