Optimal Ordering Policy with Inspection Errors and Learning Curve Consideration on Imperfect Quality Items

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Abstract

This paper develops an economic ordering policy for imperfect quality items considering the effects of learning curve and inspection errors simultaneously. Many a times, companies' efforts in quality improvement lead to learning effects in the quality of the products. Thus, in this paper we have observed the effect of learning on the percentage defectives of the shipments received from the supplier. The items received from the supplier undergoes a 100% inspection process but the inspection process may commit two types of errors, namely, Type I error and Type II error. In this scenario, some of the defective items may get passed on to the customer, which are later received back from the market. The considered mathematical model aims at maximizing the net profit obtained through the sales of both perfect and imperfect quality items while incurring various costs. Finally, we have included a numerical example to demonstrate the applicability of the proposed model.

Key words: Inventory, Imperfect quality, Learning Curve, Inspection.

1 Introduction

Traditionally, inventory models were developed based on the assumption that the items are of perfect quality. However, product quality may not be always perfect. A proportion of the produced/ordered items can be found to be defective. This aspect has received attention from many researchers. Many authors have addressed the issue of lot sizing decision for imperfect quality items. Initially, Rosenblatt and Lee [12] considered that the presence of defective items resulted in smaller lot sizes. Porteus [11] considered an imperfect production process with significant relationship between quality and lot size. Schwaller [14] extended EOQ model by assuming that the incoming lots contain a fixed proportion of defectives and derived policy using fixed and variable inspection costs. Zhang and Gerchak [17] developed a joint lot sizing and inspection policy for an EOQ model where the lot was assumed to contain a random proportion of defective units. They considered a model where the defective units cannot be used and thus must be replaced by non-defective ones. Recently, Salameh and Jaber [13] developed an EOQ model considering the lot to contain a random fraction of imperfect quality items with a known probability distribution. They considered that the received lot undergoes a 100% inspection process and at the end of the inspection, defective items are sold as a single batch. Several researchers have extended the work of Salameh and Jaber [13]. Cárdenas-Barrón [2] observed a minor correction for the expression of optimal lot size.

Goyal and Cárdenas-Barrón [4] presented a simple approach which approximately determines the order quantity when a random proportion of units are defective. Papachristos and Konstantaras [10] examined the assumptions made for avoiding shortages in S & J's model. Recently, Khan et al. [8] extended the work of S & J by considering inaccuracy in the inspection process. They considered that an inspector, while classifying the items as defective and non-defectives could make misclassifications with fixed rates.

Moreover, many researchers have investigated the effect of learning curve. A learning curve is the phenomenon of improvement in the performance due to repetitions. It was introduced in 1936 by T.P Wright in an article "Factors affecting the cost of airplanes" in the Journal of Aeronautical Sciences. Wright is believed to be the first to quantify the learning curve function. He gave the power form to the learning curve model.

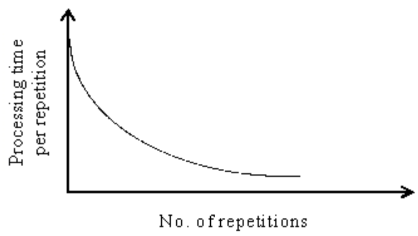

Fig.1. Wright's learning curve

Wright's learning curve is the most widely used and accepted and it has also been found to fit the empirical data quite appropriately in the studies done by Yelle [16] , Lieberman [9] etc. Thereafter, several authors have studied the effects of learning in inventory systems. Keachie and Fontana [7] examined the effects of learning on optimal lot size. Chand [3] studied the effects of learning in set-ups and process quality. Jaber and Bonney [5] extended the work of Chand [3] by assuming learning and forgetting to occur simultaneously in set-ups and process quality. Badiru [1] performed a multivariate analysis of the effect of learning and forgetting on product quality. He stated that learning affects worker performance, which ultimately can affect product quality. Recently, Jaber et al., [6] investigated the effects of learning in product quality for developing an economic production quantity model.

The author's survey of the relevant literature reveals that there is no published work that investigates the effect of learning in product quality and inspection errors simultaneously. Therefore, this paper develops an inventory model to determine the optimal order quantity in the presence of inspection errors and learning in product quality. This paper is organized as follows: 1. Introduction; 2. Notations and assumptions; 3. Mathematical model; 4. Concavity analysis of function 5. Numerical example; 6. Conclusion and 7. References.

2 Notations and Assumptions

The relevant assumptions and notations used to develop mathematical model in this paper are:

2.1 Assumptions

1. The demand rate is deterministic and constant.
2. Shortages are not allowed.
3. Lead time is negligible.
4. Replenishment is instantaneous

5. The received lot contains imperfect quality items and thus is screened for separating defective and non-defective items through a 100% inspection process.
6. The items found defective are sold at a discounted price.
7. The percentage of defectives per lot follows the Wright's learning curve.

2.2 Notations

d	Demand rate (units / unit time)
y_i	Order size of i^{th} cycle
c	Unit purchase cost
K	Ordering cost per order
h	Holding cost per unit per unit time
$p(i)$	Percentage of defectives in i^{th} shipment
E_1	Probability of Type I error
E_2	Probability of Type II error
β_{1i}	Effective rejection rate of i^{th} cycle
β_{2i}	Effective return rate of i^{th} cycle
x	Screening rate
c_s	Unit cost of inspection
c_a	Unit cost of accepting a defective item
c_r	Unit cost of rejecting a non-defective item
s	Unit selling price of non-defective items
v	Unit selling price of defective items $(v < s)$
T_i	Cycle time of i^{th} cycle
t_{1i}	Inspection time of i^{th} cycle
t_{2i}	Remaining time of i^{th} cycle $(= T_i - t_{1i})$

3 Formulation of the Mathematical Model

Consider a lot of size y_i received from the supplier in the beginning of i^{th} cycle. Each received lot contains $p(i)$ percentage defectives and thus it is subjected to a 100% inspection process. This inspection is carried out so that the defective items can be removed from the lot and sold at a discounted price as a single batch. When the lot is screened for the defective items, two types of error may be committed, namely Type I error and Type II error. Let E_1 be the probability of committing Type I error, i.e. probability of rejecting a non-defective item during inspection and let E_2 be the probability of committing Type II error, i.e. probability of accepting a defective item. Therefore, the number of items rejected from the lot of size y_i is the sum of incorrectly rejecting a non-defective item and correctly rejecting a defective item. We have,

Number of defective items in the i^{th} lot, $N_{d\,i} = y_i(1 - p(i))$;

Number of non-defectives in the i^{th} lot, $N_{nd\,i} = y_i p(i)$;

Number of items rejected from the i^{th} lot, $N_{r\,i} = E_1 y_i(1 - p(i)) + (1 - E_2) y_i p(i)$;

Therefore, the effective rejection rate of i^{th} cycle is,

$\beta_{1i} = \dfrac{N_{ri}}{y_i} = E_1(1 - p(i)) + (1 - E_2)\,p(i)$;

The items rejected during the inspection process are sold as a single batch at a discounted price of v per unit. Also due to inspection error, some of the defective items are identified as non-defectives during inspection and thus get passed on to the customers. These items are later returned from the market and sold as a single batch at a discounted price at the end of the cycle.

So, we have

Number of defective items not screened in inspection, $N_{mi} = E_2 y_i p(i)$;

Therefore, effective return rate of i^{th} cycle is, $\beta_{2i} = \dfrac{N_{mi}}{y_i} = E_2\, p(i)$;

The behaviour of the inventory levels in a cycle is shown in figures 2(a), 2(b) and 2(c). The lot for i^{th} cycle is received at time zero, then till time t_{1i} the inventory is classified into defectives and non-defectives while serving the demand from the items classified as non-defective. Here, it is reasonable to assume that the rate of inspection is greater than the demand rate $(x > d)$. Also, due to inspection error, some of the defective items that are sold to the customers get returned by the market as shown in fig. 2(c). These returned items are replaced with non-defective ones. Thus, in an i^{th} cycle after time t_{1i} until the end of the cycle, inventory level as shown in fig. 2(a) decreases due to demand and the replacement of returned items.

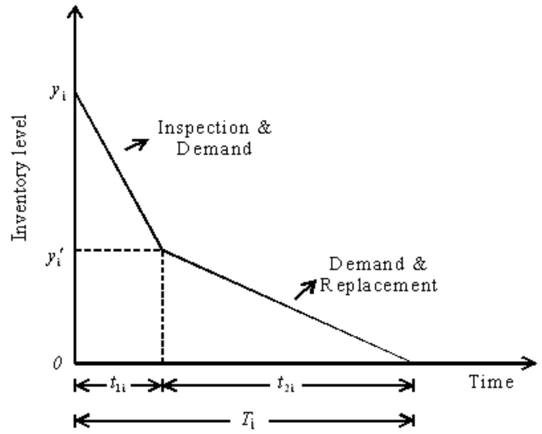

Fig.2(a). The inventory level of the i^{th} shipment

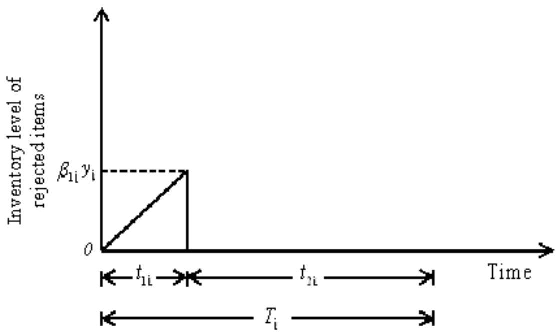

Fig. 2(b). The inventory level of rejected items of i^{th} cycle

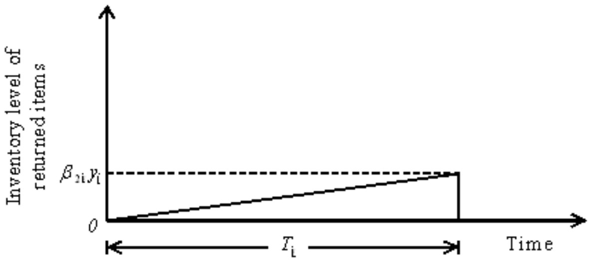

Fig. 2(c). The inventory level of items returned during i^{th} cycle

In this paper, the learning effect on the percentage of defectives per lot is assumed to follow Wright's power function formula and can be expressed as $p(i) = p_0 i^{-L}$ where p_0 is the initial percentage of defectives, L is the learning slope in the Wright's formulation for the learning curve $(0 \le L < 1)$ and i is the cumulative number of shipments. This form of learning curve for percentage defectives has earlier been described by Jaber [6] and found to be appropriate empirically for the learning phase of the learning curve.

To avoid shortages, it is reasonable to assume that the number of items inspected as non-defective is at least equal to the sum total of the actual demand and the number of units used to replace the returned ones, i.e.

$$y_i(1-\beta_{1i}) \ge dT_i + y_i\beta_{2i} \Rightarrow y_i(1-(\beta_{1i}+\beta_{2i})) \ge dT_i$$

For the limiting case in each cycle, we have

$$y_i(1-(\beta_{1i}+\beta_{2i})) = dT_i \Rightarrow T_i = \frac{y_i(1-(\beta_{1i}+\beta_{2i}))}{d} \tag{1}$$

The total revenue for an i^{th} cycle is the sum total of revenue generated from the sales of non-defectives and defectives and is given by

$$R_i = sy_i(1-\beta_{1i}) + vy_i(\beta_{1i}+\beta_{2i}) \tag{2}$$

and the total cost for an i^{th} cycle is the sum total of ordering cost, purchase cost, inspection cost, error costs and the holding cost and is given by

$$C_i = K + cy_i + c_s y_i + c_r y_i(\beta_{1i}+\beta_{2i} - p(i)) + c_a y_i \beta_{2i}$$

$$+ h\left\{ \underbrace{\frac{(y_i - y_i')t_{1i}}{2} + y_i' t_{1i} + \frac{y_i' t_{2i}}{2}}_{\text{Refer to fig 1(a)}} + \underbrace{\frac{y_i\beta_{1i}t_{1i}}{2}}_{\text{Refer to fig 1(b)}} + \underbrace{\frac{y_i\beta_{2i}T_i}{2}}_{\text{Refer to fig 1(c)}} \right\}$$

where $y_i' = y_i - y_i\beta_{1i} - dt_{1i} \Rightarrow y_i - y_i' = y_i\beta_{1i} + dt_{1i}$; $t_{1i} = \frac{y_i}{x}$ and $t_{2i} = T_i - t_{1i}$.

$$\Rightarrow C_i = K + cy_i + c_s y_i + c_r y_i(\beta_{1i}+\beta_{2i} - p(i)) + c_a y_i \beta_{2i}$$

$$+ \frac{h}{2}\left\{ \frac{y_i^2(1+\beta_{1i})}{x} + y_i(1-\beta_{1i}+\beta_{2i})T_i - \frac{dy_iT_i}{x} \right\} \tag{3}$$

Thus, the total profit per i^{th} cycle is

$$TP_i = R_i - C_i$$

$$= sy_i(1-\beta_{1i}) + vy_i(\beta_{1i}+\beta_{2i}) - K - cy_i - c_s y_i - c_r y_i(\beta_{1i}+\beta_{2i} - p(i)) \tag{4}$$

$$- c_a y_i\beta_{2i} - \frac{h}{2}\left\{ \frac{y_i^2(1+\beta_{1i})}{x} + y_i(1-\beta_{1i}+\beta_{2i})T_i - \frac{dy_iT_i}{x} \right\}$$

Here, for an infinite planning horizon, i shall be regarded as an input parameter, thereby leaving the total profit function TP_i a function of variable y_i. Hence, our objective is to obtain the optimal value of y_i that maximizes the net profit.

Now, the total profit per unit time for an i^{th} cycle is given by

$$TPU\ (y_i) = \frac{R_i - C_i}{T_i} \text{ where } T_i = \frac{y_i(1-(\beta_{1i}+\beta_{2i}))}{d};$$

$$\begin{aligned}\therefore TPU\ (y_i) = & \frac{d}{[1-(\beta_{1i}+\beta_{2i})]}\left\{ s(1-\beta_{1i}) + v(\beta_{1i}+\beta_{2i}) - \frac{K}{y_i} - c - c_s - c_r(\beta_{1i}+\beta_{2i}-p(i)) - c_a\beta_{2i} \right\} \\ & - \frac{hy_i}{2[1-(\beta_{1i}+\beta_{2i})]}\left\{ \frac{d(2\beta_{1i}+\beta_{2i})}{x} + \left[(1-\beta_{1i})^2 - \beta_{2i}{}^2\right] \right\}\end{aligned} \quad (5)$$

The total profit per unit time is a concave function of y_i, as

$$\frac{dTPU\ (y_i)}{dy_i} = \frac{Kd}{y_i{}^2[1-(\beta_{1i}+\beta_{2i})]} - \frac{h}{2[1-(\beta_{1i}+\beta_{2i})]}\left\{ \frac{d(2\beta_{1i}+\beta_{2i})}{x} + \left[(1-\beta_{1i})^2 - \beta_{2i}{}^2\right] \right\} \quad (6)$$

$$\Rightarrow \frac{d^2TPU\ (y_i)}{dy_i{}^2} = \frac{-2Kd}{y_i{}^3[1-(\beta_{1i}+\beta_{2i})]} < 0 \qquad \forall\ y_i > 0$$

Note that, $0 < 1-(\beta_{1i}+\beta_{2i}) \le 1$ as $1-(\beta_{1i}+\beta_{2i}) = (1-E_1)(1-p(i))$.

Therefore, the necessary condition for $TPU\ (y_i)$ to be maximum is $\frac{dTPU\ (y_i)}{dy_i} = 0$.

Hence, the optimal value of y_i can be obtained by differentiating the function $TPU\ (y_i)$ w.r.t. y_i and equating the result to zero, i.e. by setting $\frac{dTPU\ (y_i)}{dy_i} = 0$.

$$\Rightarrow \quad y_i{}^* = \sqrt{\frac{2Kd}{h\left\{ \frac{d(2\beta_{1i}+\beta_{2i})}{x} + \left[(1-\beta_{1i})^2 - \beta_{2i}{}^2\right] \right\}}} \quad \text{for each } i \quad (7)$$

where $\beta_{1i} = E_1(1-p(i)) + (1-E_2)p(i)$; $\beta_{2i} = E_2 p(i)$ and $p(i) = p_0 i^{-L}\ (0 \le L < 1)$

Now, let us consider a finite planning horizon of n cycles in which at the beginning of each cycle, a lot of fixed size y is received from the supplier and $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} y = ny = Q(say)$. Then the total revenue and the total cost functions over the finite planning horizon are given by

$$\sum_{i=1}^{n} R_i = sQ - \frac{Q}{n}(s-v)\sum_{i=1}^{n}\beta_{1i} + v\frac{Q}{n}\sum_{i=1}^{n}\beta_{2i} \quad (8)$$

$$\begin{aligned}\sum_{i=1}^{n} C_i = & nK + (c+c_s)Q + c_r.\frac{Q}{n}\left\{\sum_{i=1}^{n}\beta_{1i} - \sum_{i=1}^{n}p(i)\right\} + (c_r+c_a).\frac{Q}{n}\sum_{i=1}^{n}\beta_{2i} \\ & + \frac{h}{2d}.\frac{Q^2}{n^2}\left\{\frac{d}{x}\sum_{i=1}^{n}(2\beta_{1i}+\beta_{2i}) + \sum_{i=1}^{n}\left[(1-\beta_{1i})^2 - \beta_{2i}{}^2\right]\right\}\end{aligned} \quad (9)$$

From $\beta_{1i} = E_1(1-p(i)) + (1-E_2)p(i)$; $\beta_{2i} = E_2 p(i)$ and $p(i) = p_0 i^{-L}\ (0 \le L < 1)$

we can obtain, $\sum_{i=1}^{n}\beta_{1i} = nE_1 + (1-E_1-E_2)\sum_{i=1}^{n}p(i)$; $\sum_{i=1}^{n}\beta_{2i} = E_2\sum_{i=1}^{n}p(i)$ and

$$\sum_{i=1}^{n} p(i) = \sum_{i=1}^{n} p_0 i^{-L} \cong \int_0^n p_0 u^{-L} du = \frac{p_0 n^{1-L}}{1-L} \quad (10)$$

Similarly, we can also obtain

$$\sum_{i=1}^{n}\left[(1-\beta_{1i})^2-\beta_{2i}^{\;2}\right]=n(1-E_1)^2-2(1-E_1)(1-E_1-E_2)\sum_{i=1}^{n}p(i)+(1-E_1)(1-E_1-2E_2)\sum_{i=1}^{n}p^2(i)$$

and $\sum_{i=1}^{n}p^2(i)=\dfrac{p_0^{\;2}.n^{1-2L}}{1-2L}$.

Then Equations (8) and (9) reduces to

$$\sum_{i=1}^{n}R_i = sQ(1-E_1)+vQE_1-Q\{(s-v)(1-E_1)-sE_2\}.\frac{p_0n^{-L}}{1-L} \tag{11}$$

and

$$\sum_{i=1}^{n}C_i = nK+(c+c_s)Q+c_rQE_1\left(1-\frac{p_0n^{-L}}{1-L}\right)+c_aQE_2.\frac{p_0n^{-L}}{1-L}+\frac{h}{2d}.\frac{Q^2}{n}\left\{(1-E_1)^2+\frac{2E_1d}{x}\right.$$
$$\left.+\left[\frac{d}{x}(2-2E_1-E_2)-2(1-E_1)(1-E_1-E_2)\right].\frac{p_0n^{-L}}{1-L}+(1-E_1)(1-E_1-2E_2)\frac{p_0^{\;2}n^{-2L}}{1-2L}\right\} \tag{12}$$

The total profit over n cycles is given by $\sum_{i=1}^{n}TP_i=\sum_{i=1}^{n}R_i-\sum_{i=1}^{n}C_i$ which is a function of two variables Q and n, where n being the number of replenishments is a discrete variable. Moreover, $\sum_{i=1}^{n}TP_i$ is an increasing function of Q and concave in n (Refer to Theorem A in section 4), so we find the optimal number of shipments n^* that maximizes the total profit function at a fixed value of Q, say Q_0 by searching for n^* such that $\sum_{i=1}^{n^*-1}TP_i(Q_0,n^*-1)<\sum_{i=1}^{n^*}TP_i(Q_0,n^*)$ and $\sum_{i=1}^{n^*+1}TP_i(Q_0,n^*+1)<\sum_{i=1}^{n^*}TP_i(Q_0,n^*)$.

4 Concavity Analysis of the Total Profit Function over n Cycles

Theorem A: The total profit function, $\sum_{i=1}^{n}TP_i(Q_0,n)$ is a concave function of n.

Proof: To simplify the analysis and without loss of generality, let us assume that the function $\sum_{i=1}^{n}TP_i(Q_0,n)$ is a continuous and differentiable function over n and n is a real number, $n>0$.

$$\sum_{i=1}^{n}TP_i(Q_0,n)=[s(1-E_1)+vE_1-c-c_s-c_rE_1].Q_0-nK$$
$$-[(s-v)(1-E_1)-sE_2-c_rE_1+c_aE_2].Q_0.\frac{p_0n^{-L}}{1-L}-\frac{h}{2d}.\frac{Q_0^{\;2}}{n}\left\{(1-E_1)^2+\frac{2E_1d}{x}\right.$$
$$\left.+\left[\frac{d}{x}(2-2E_1-E_2)-2(1-E_1)(1-E_1-E_2)\right].\frac{p_0n^{-L}}{1-L}+(1-E_1)(1-E_1-2E_2)\frac{p_0^{\;2}n^{-2L}}{1-2L}\right\}$$
$$\tag{13}$$

$$\frac{d\left(\sum_{i=1}^{n} TP_i(Q_0,n)\right)}{dn} = -K + \left[(s-v)(1-E_1) - sE_2 - c_r E_1 + c_a E_2\right].\frac{Q_0}{n}.\left(\frac{L}{1-L}\right).p_0 n^{-L}$$

$$+\frac{h}{2d}.\frac{Q_0^2}{n^2}\left\{(1-E_1)^2 + \frac{2E_1 d}{x} + \left[\frac{d}{x}(2-2E_1-E_2)\right.\right.$$

$$\left.\left. -2(1-E_1)(1-E_1-E_2)\right].\left(\frac{1+L}{1-L}\right).p_0 n^{-L} + (1-E_1)(1-E_1-2E_2).\left(\frac{1+2L}{1-2L}\right).p_0^2 n^{-2L}\right\} \tag{14}$$

and

$$\frac{d^2\left(\sum_{i=1}^{n} TP_i(Q_0,n)\right)}{dn^2} = -\left[(s-v)(1-E_1) - sE_2 - c_r E_1 + c_a E_2\right].\frac{Q_0}{n^2}.\left(\frac{L(1+L)}{1-L}\right).p_0 n^{-L}$$

$$-\frac{h}{d}.\frac{Q_0^2}{n^3}\left\{(1-E_1)^2 + \frac{2E_1 d}{x} + \frac{1}{2}.\left[\frac{d}{x}(2-2E_1-E_2) - 2(1-E_1)(1-E_1-E_2)\right].\right.$$

$$\left.\left(\frac{(1+L)(2+L)}{1-L}\right).p_0 n^{-L} + (1-E_1)(1-E_1-2E_2)\left(\frac{(1+L)(1+2L)}{1-2L}\right).p_0^2 n^{-2L}\right\} \tag{15}$$

Now, in order to obtain the optimal value of n, we set $\dfrac{d\left(\sum_{i=1}^{n} TP_i(Q_0,n)\right)}{dn} = 0$

$$\Rightarrow \left[(s-v)(1-E_1) - sE_2 - c_r E_1 + c_a E_2\right].\frac{Q_0}{n}.\left(\frac{L}{1-L}\right).p_0 n^{-L}$$

$$= K - \frac{h}{2d}.\frac{Q_0^2}{n^2}\left\{(1-E_1)^2 + \frac{2E_1 d}{x} + \left[\frac{d}{x}(2-2E_1-E_2)\right.\right.$$

$$\left.\left. -2(1-E_1)(1-E_1-E_2)\right].\left(\frac{1+L}{1-L}\right).p_0 n^{-L} + (1-E_1)(1-E_1-2E_2)\left(\frac{1+2L}{1-2L}\right).p_0^2 n^{-2L}\right\} \tag{16}$$

Multiplying both sides of Eq. (16) by $\dfrac{(1+L)}{n}$ and substituting the value of its L.H.S in Eq. (15) we get,

$$\frac{d^2\left(\sum_{i=1}^{n} TP_i(Q_0,n)\right)}{dn^2} = -\frac{(1+L)K}{n} - \frac{h}{2d}.\frac{Q_0^2}{n^3}\left\{\left[(1-E_1)^2 + \frac{2E_1 d}{x}\right].(1-L)\right.$$

$$+\left[\frac{d}{x}(2-2E_1-E_2) - 2(1-E_1)(1-E_1-E_2)\right].\left(\frac{1+L}{1-L}\right).p_0 n^{-L} \tag{17}$$

$$\left.+(1-E_1)(1-E_1-2E_2)\left(\frac{(1+L)(1+2L)}{1-2L}\right).p_0^2 n^{-2L}\right\}$$

Since $0 \le L < 1$, $0 \le E_1 < 1$, $0 \le E_2 < 1$ and $n > 0$ we conclude that

$\frac{d^2\left(\sum_{i=1}^{n} TP_i(Q_0, n)\right)}{dn^2} < 0$ at the optimal value of n that can be obtained from equation (16) provided $L \neq \frac{1}{2}$. Hence, $\sum_{i=1}^{n} TP_i(Q_0, n)$ is a concave function of n.

Also, based on the parametric values given in the numerical example, the following graph depicts the concavity of the function $\sum_{i=1}^{n} TP_i(Q_0, n)$.

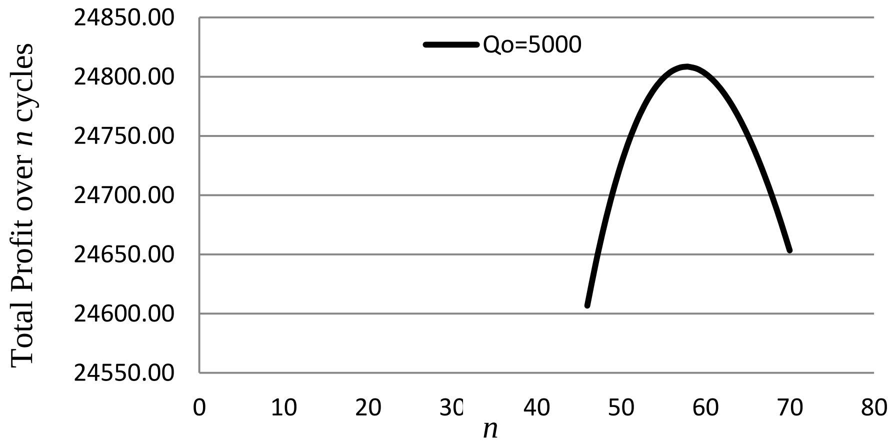

Fig.3. Concavity of the Total Profit Function

5 Numerical Example

5.1 Example 1. Let $K = \$100$, $d = 40{,}000$ units per year, $x = 87{,}600$ units per year, $h = \$5$ per unit per year, $s = \$50$, $v = \$20$, $c = \$25$, $c_s = \$2$, $c_r = \$40$, $c_a = \$80$, $p_0 = 0.35$, $E_1 = 0.2$, $E_2 = 0.3$ and $L = 0.4$. Then for an infinite planning horizon, using equations (7) and (5), the percentage defectives per cycle, the optimal lot size and the corresponding net profit per unit time is given as follows:

Cycle no. (i)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
p(i)	**0.35**	**0.265**	**0.226**	**0.201**	**0.184**	**0.171**	**0.161**	**0.152**
y*ᵢ	**1441.49**	**1433**	**1428.35**	**1425.28**	**1423.04**	**1421.3**	**1419.89**	**1418.73**
TPU*(yᵢ)	**8558.05**	**151696.09**	**207985.08**	**239939.18**	**261169.4**	**276581.95**	**288427.99**	**297902.91**

5.2 Example 2. Given the same parametric values as mentioned in example 1, consider a finite planning horizon in which a fixed number of $Q_0 = 5000$ units are to be delivered in n equal lot sizes. By applying the computational procedure mentioned at the end of section 3, we get $n^* = 57.78 \approx 58$ as the optimal number of replenishments and the total profit over n cycles is then given by $\sum_{i=1}^{n^*} TP_i(Q_0, n^*) = \$24808.47$.

6 Conclusion

In this paper, we developed a mathematical model to determine the optimal order quantity of imperfect quality items which are subjected to 100% inspection on their arrival, but the inspection process is not error-free. Moreover, we also investigated the effect of learning on percentage of defectives in each lot. To examine the behaviour of learning, power function formula of Wright's learning curve model has been used. In this study, it has been found that the influence of learning and inspection errors is significant in determining the optimal lot size.

7 References

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