Network Design Model with Evacuation Constraints

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Abstract

In recent years terrorism activities have been increasing in scope worldwide as well as the global warming process has had a direct impact on the weather in various climates, subjecting countries around the world to unusually severe storms. Thus for the policy maker it is not only a matter of network design (roads and facilities) in terms of costs and level of service for commuting, but also the matter of network design in terms of costs and evacuation time. This paper focuses on the later – the development of a model for the design of an optimal network in terms of minimizing both evacuation time and network constructions costs. However, the optimal model's complexity does not allow solution within a reasonable timeframe. Therefore, a fast heuristic model was developed based on the minimum-cost algorithm, using the unimodular properties of the model for obtaining integral results. The heuristic algorithm was compared to the optimal algorithm (both based on ILOG CPLEX) on various network scenarios and produced on average 10% higher construction costs than the optimal algorithm. On the other hand, the execution time of the heuristic algorithm was significantly faster than the optimal algorithm.

Key words: Evacuation, Network Design, Facility Location, Optimization, Heuristics.

1 Introduction

In recent years terrorism activities have been increasing in scope worldwide, with the events of 9/11 constituting a significant turning point in the global perception of security needs. The idea that terrorists have gained global reach enabling them to strike anywhere in the world, at any given time, has raised the level of alertness and readiness. Concurrently, the global warming process has had a direct impact on the weather in various climates, subjecting countries around the world to unusually severe storms. Hurricane Katrina, which had nearly wiped out the city of New Orleans, the cyclone storm in Myanmar, and the earth quake in Haiti has been painful reminders of humanity's frailty in the face of the forces of nature.

Such recent events have taught us that in order to reduce damages and minimize casualties we must prepare for the worst. It is our responsibility to design our facilities and infrastructure accordingly, both in terms of access routes and in terms of location, in order to be prepared to evacuate them quickly at short notice.

Thus, for the policy maker it is not only a matter of network design in terms of costs and level of service for commuting, but also the matter of network design in terms of costs and evacuation time, as it is imperative to minimize casualties with quick and efficient evacuation, as well as to utilize the infrastructure for logistic support (Sheu, 2007).

Evacuation planning can be related to the network design problem, the facility location problem, and network flow models. The network design problem (Magnanti and Wong, 1984) is a set of problems designed to construct networks with different objective functions in mind, given the flow can be served by a network constrained by capacity. Facility location problems (Nagy and Salhi, 2007), aims at locating a set of facilities, both serving and being served, in a network, in order to achieve an objective function with a set of constraints (Avella and Boccia, 2009). Models that combine facility location and vehicle routing problems (Bozkaya et al., 2010, Yi and Özdamar, 2007) were also developed, as clearly integrating two interrelated models together can increase efficiency and reduce costs of distribution systems.

Most evacuation planning models are dealing with predetermined networks, such as the model by Sherali et al. (1991), a location-allocation model that minimizes evacuation time but disregards costs, or a model by Xie et al. (2010) that increases network capacity for evacuation by lane reversal and optimizing crossing. Other models were developed to locate relief facilities in a known network (Balcik and Beamon, 2008), but none were found to suggest the structure of the network. Network flow models, such as the maximum-flow and minimum-cost problems (Hillier and Lieberman, 2005) are well known problems that find the total flow from origin to destination (the former), or the minimal cost for flow from origin to destination, given costs associated with arcs and nodes (the later). These models assume costs per unit, rather than construction costs associated with network design problems and facility location problems.

This paper focuses on the development of a multi-objective model for the design of an optimal network in terms of minimizing both evacuation time and network constructions costs. Multi-objective models (Coello Coello et al., 2002) are dealing with models that optimize problems with more than one objective function, for example, modeling delivery system with three objective functions: cost, time, and satisfaction (Tzeng et al., 2007), or optimizing both efficiency and equity measures for school-bush routing (Bowerman et al., 1995).

The paper is organized as follows: a multi-objective evacuation network design problem is introduced: an optimal formulation (section 2), a simplified, single objective model (section 3), and an efficient heuristic based on the minimum-cost problem (section 4). Section 5 provides evaluation of the heuristic algorithm, and section 6 presents conclusions.

2 Network evacuation design problem

Assume a network, as illustrated in Figure 1, with: a) set of origin nodes designed to serve as warehouses, facilities, populated districts, stadium sections, etc., each with estimated construction cost and capacity, b) set of destination nodes designed to serve as evacuation areas (assembly areas, shelters, safe zones, etc., each with estimated construction cost and capacity as well, and c) set of possible transportation infrastructure (roads, aisles, pathways, etc.), each with estimated construction cost and capacity. We are looking for a recommendation for the location of origin nodes, destination nodes, and a network connecting those nodes that will minimize the evacuation time with minimal costs, taking into account the total demand (population, commodities, military mobile-resources, spectators, etc.).

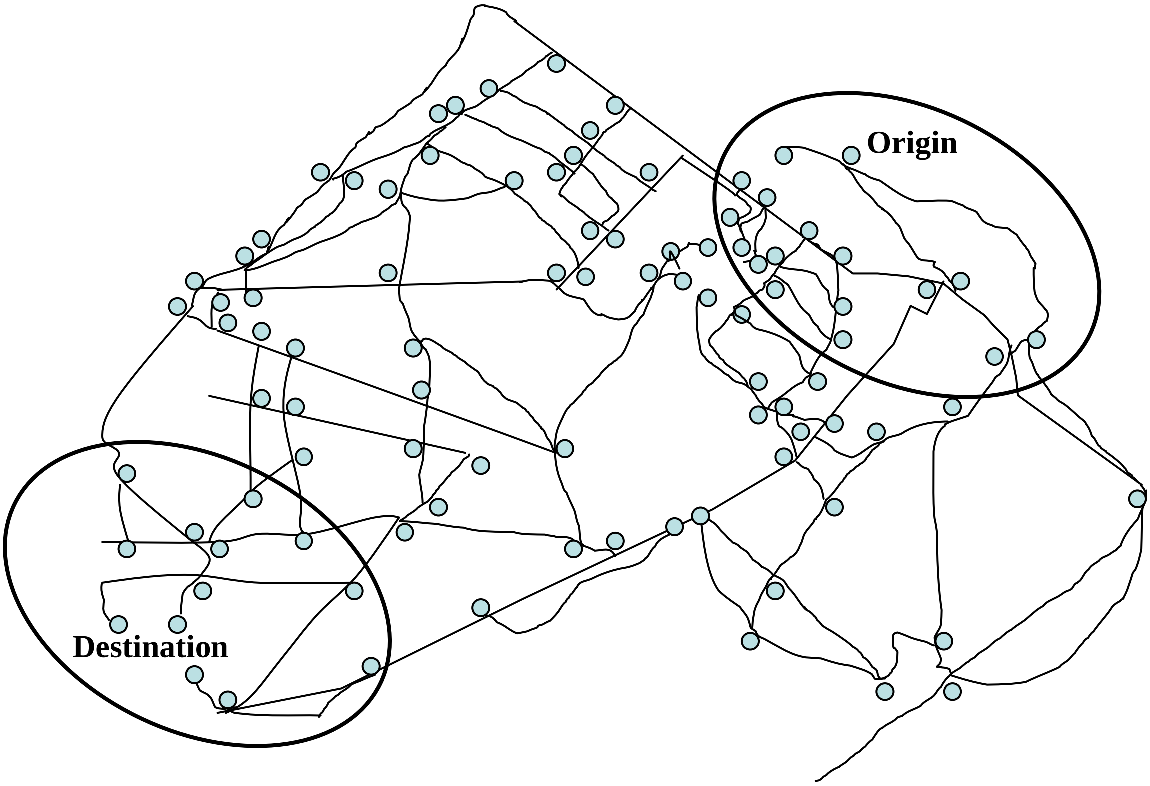

Figure 1 - Illustrative evacuation network

Thus, the Network Evacuation Design Problem (NEDP) can be formulated as follows.

Let $G(N,A)$ be a graph, with N nodes and A arcs, $\{O\}\in N$ origin candidate set (demand), $\{D\}\in N$ destination candidate set (supply). Also let $\{(i,j)\}\in A$ arc candidate set, with $i,j\in[1\ldots N]$.

Parameters

$Ua_{i,j}$ = capacity of arc (i,j).

Un_i = the capacity of node i.

$Ca_{i,j}$ = construction cost of arc *(i,j)*.

Cn_i = construction cost of node *i*.

TD = total demand

Decision variables

$f_{i,j}$ = flow along arc *(i,j)*

b_i = quantity of demand allocated to node i (positive value – supply, negative value – demand).

T = evacuation time.

Model NEDP1

(1) $$\text{Minimize} \sum_{(i.j)\in A} Ca_{i,j} \cdot Ya_{i,j} + \sum_{i\in N} Cn_i \cdot Yn_i$$

(2) $$\text{Minimize } T$$

Subject To

(3) $$0 \le b_i \le Un_i \cdot Yn_i \quad \forall i \in O$$

(4) $$0 \le -b_i \le Un_i \cdot Yn_i \quad \forall i \in D$$

(5) $$\sum_{i\in O} b_i = TD$$

(6) $$\sum_{i\in D} b_i = -TD$$

(7) $$b_i = 0 \quad \forall i \notin O \cup D$$

(8) $$f_{i,j} \le Ua_{i,j} \cdot Ya_{i,j} \cdot T \quad \forall (i,j) \in A$$

(9) $$\sum_{j=1}^{n} f_{i,j} - \sum_{j=1}^{n} f_{i,j} = b_i \quad \forall i$$

(10) $$f_{i,j} \ge 0,\ f_{i,j} \in \mathbb{Z} \quad \forall (i,j) \in A$$

(11) $$Ya_{i,j} \in \{0,1\} \quad \forall (i,j) \in A$$

(12) $$Yn_i \in \{0,1\} \quad \forall i \in N$$

(13) $$T > 0$$

Objectives (1) and (2) represent the construction costs and evacuation time respectively. Constraints (3) and (4) restrict demand to facility capacity, constraints (5) and (6) enforce that total demand is met, constraint (7) defines transshipment nodes, constraint (8) defines arcs' capacity over time, constraint (9) defines conservation of flow, constraint (10) defines integral flow, constraints (11) and (12) define binary variables, and constraint (13) enforces positive evacuation time.

However, the NEDP1 complexity does not allow solution within a reasonable timeframe due to: a) Multi-objective problem: solution cost and evacuation time, b) The use of integer variables, and c) Integral flow.

Therefore, in order to decrease complexity, a single objective algorithm was constructed, as well as a heuristic Model that is based on the minimum-cost algorithm.

3 Single objective model

Multi-objective problems are usually difficult to solve (Coello Coello, Van Veldhuizen and Lamont, 2002, Current et al., 1990), thus it is sought, if possible, to construct a single objective function. The NEDP1 is unsuitable for that, as it is impossible to convert time into costs. On the other hand, decision variable T can be isolated, as it is relevant only to constraint (8). Thus an iterative algorithm was constructed by removing objective function (2), substituting constraint (13) with

(14) $T = T'$

We define NEDP2 as a single objective variation of the model with T' as an input value. Then, given a predetermined set of possible evacuation times $\{T'\}$ and an optimal costs set $\{C\}$, resulted from executing ENDP2 $\left|\{T'\}\right|$ time it is possible to construct the pareto front for NEDP1, as illustrated in Figure 2.

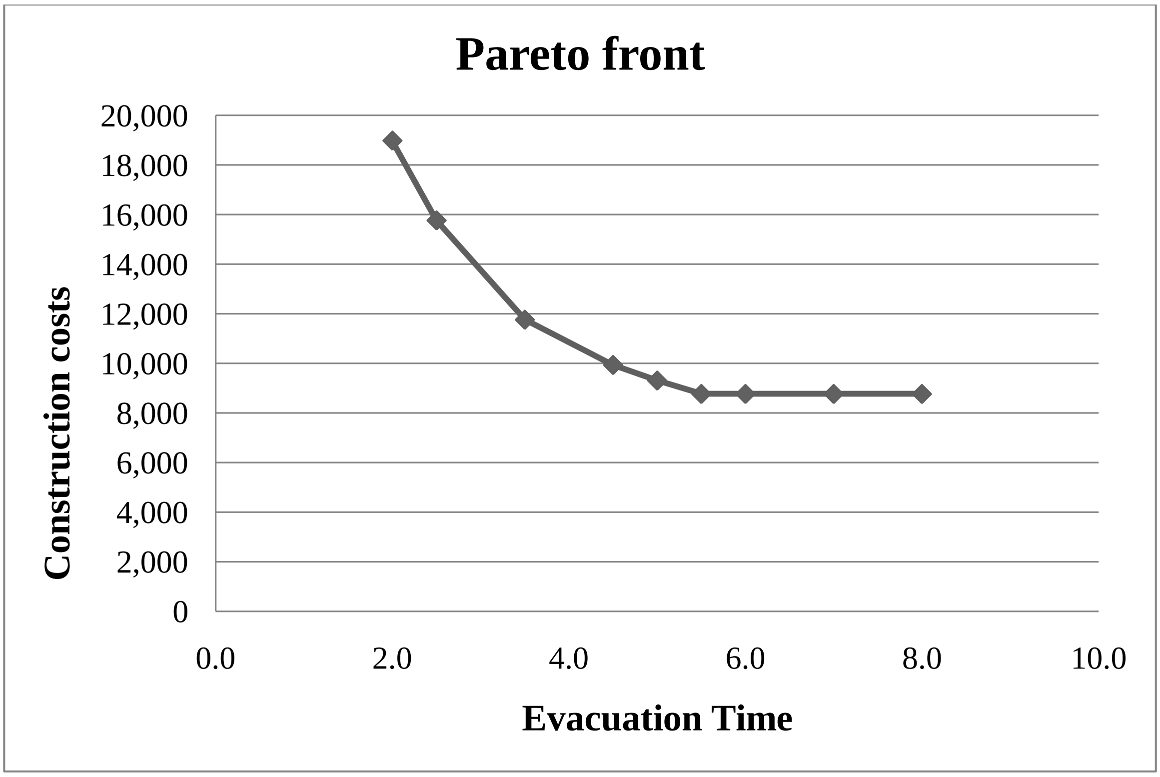

Figure 2 - Pareto front resulted from NEDP2

Even though the NEDP2 model is single objective, the MIP properties retain its NP-complete properties(Schrijver, 1986).

4 Minimum-cost heuristic model

In order to cope with the complexity of the model an efficient, reliable and quickly resolvable heuristic algorithm was developed – NEDP3. The heuristic model is based on the minimum-cost problem which was researched extensively in the past and as a result many efficient algorithms were developed (Ahuja et al., 1992), and the unimodular properties of the minimum-cost problem (Schrijver, 1986, Schrijver, 2003), meaning that if all parameters (C, A, b) of the problem are integral, so is the result vector X.

In order to transform the model to a minimum-cost model, we do the following: a) *Demand and supply nodes transformation:* each possible demand or supply node i is transformed to an arc (i,i''), with $Ca_{i',i} = Cn_i, Ua_{i',i} = Un_i$. b) *Unit cost transformation:* As the cost $Ca_{i,j}$ is the construction cost of an arc rather than the unit cost $C_{i,j}$ required by the minimum-cost model, an estimated unit cost $C_{i,j}$ was calculated:

$$C_{i,j} = round\left(\frac{Ca_{i,j}}{Ua_{i,j}} \cdot 10^m\right) \tag{15}$$

Where m is the precision level, with m=2 practically sufficient.

4.1 Minimum-cost formulation

NEDP3 can be formulated as follows:

$$Minimize \sum_{(i.j)\in A} C_{i,j} \cdot f_{i,j} \tag{16}$$

$$b_i \geq 0 \quad \forall i \in O \tag{17}$$

$$b_i \leq 0 \quad \forall i \in D \tag{18}$$

$$\sum_{i\in O} b_i = TD \tag{19}$$

$$\sum_{i\in D} b_i = -TD \tag{20}$$

$$b_i = 0 \quad \forall i \notin O \cup D \tag{21}$$

$$f_{i,j} \leq U_{i,j} \cdot T \quad \forall (i,j) \in A, i \notin O \wedge j \notin D \tag{22}$$

$$f_{i,j} \leq U_{i,j} \quad \forall (i,j) \in A, i \in O \vee j \in D \tag{23}$$

$$\sum_{j=1}^{n} f_{i,j} - \sum_{j=1}^{n} f_{i,j} = b_i \quad \forall i \tag{24}$$

$$f_{i,j} \geq 0 \tag{25}$$

Where constraint (22) restrict the flow along road sections to the capacity over time, while constraint (23) restrict the flow to the facility capacity.

4.2 Heuristic algorithm:

1. $best=\infty, k=1$
2. Execute NEDP3
3. If $solution<best$ then $best=solution$, retain $\{f\}$

4. In case the flow is less than capacity, it is necessary to preserve $Ca_{i,j} = C_{i,j} \cdot f_{i,j} \quad \forall\, 0 < f_{i,j} < U_{i,j}$, thus the unit cost is updated as follows:

$$C_{i,j} = \begin{cases} round\left(\dfrac{Ca_{i,j}}{f_{i,j}} \cdot 10^m\right) & \forall\, 0 < 2 \cdot f_{i,j} < U_{i,j} \\ C_{i,j} & Otherwise \end{cases} \tag{26}$$

5. k=k+1
6. If k<(maximum number of iterations) then go to step 2, otherwise terminate.

5 Algorithm evaluation

For the evaluation of NEDP3, in terms of execution time and quality of the results, synthetic networks were generated, with the same structure, as illustrated in Figure 3, and different sizes. Assume a $n \times n$ grid of candidate bi-directional road sections, each assigned construction cost and capacity, $n \times o$ a set of possible origin nodes (nodes to evacuate), and $n \times d$ a set of possible destination nodes (evacuating areas). Both origin and destination nodes are connected to the grid with arcs that holds the construction and capacity of the nodes.

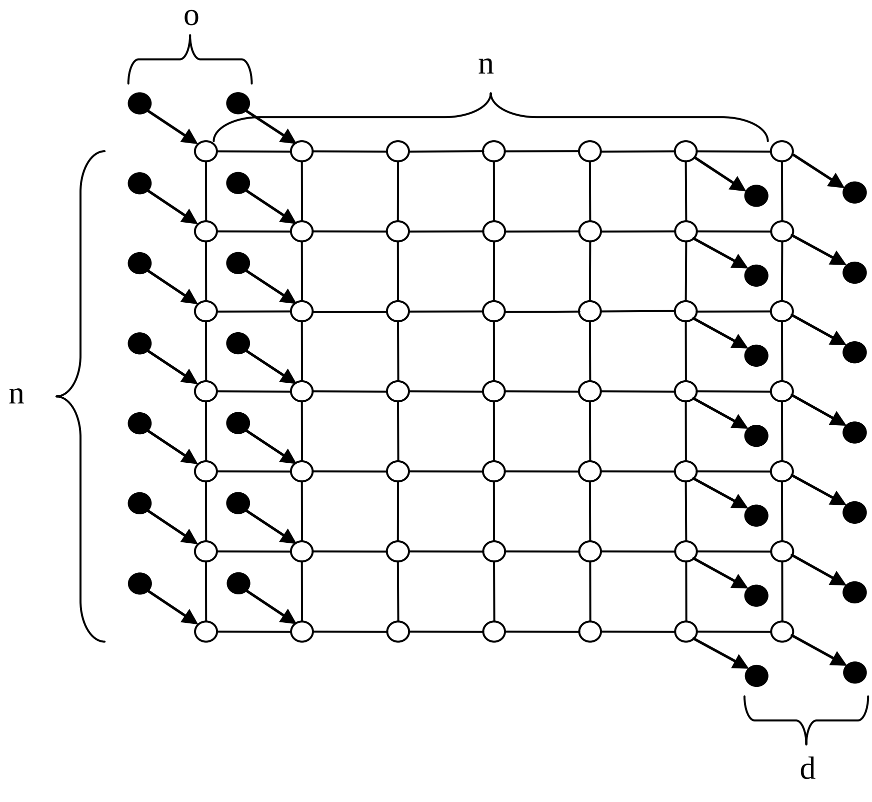

Figure 3 - Synthetic evacuation network

Executions were carried out for a single iteration of NEDP2 and NEDP3 with ILOG CPLEX 12.2 (IBM, 2010), on Intel M540 @ 2.53Ghz based computer with 4GB memory, running windows 7 64bit. The results of NEDP2 (O) versus NEDP3 (H) are summarized in Table 1. For each network the costs and execution times are presented for both models, and the gap between the optimal and heuristic solution.

"best O @ H time" provides the best solution found by NEDP2 at the time NEDP3 was terminated, while "O time @ best H" is the time it takes NEDP2 to reach a solution similar to NEDP3. These indicators are valuable as it further compares the search logic of NEDP2 with NEDP3.

Table 1 - Results of NEDP2 and NEDP3

	Cost		Execution time [hh:mm:ss]		Gap			
Nodes	**O**	**H**	**O**	**H**			**best O @ H time**	**O time @ best H**
60	9,314	10,406	00:00:01	00:00:01	12%			
140	14,598	14,861	00:00:07	00:00:01	2%			
2,700	124,097	128,823	01:56:35	00:00:06	4%	*	125,458	00:00:05
10,400	245,225	249,868	03:03:00	00:00:29	2%	*	581,150	01:20:00
40,800	991,517	988,926	03:09:00	00:02:39	0%	*	47,604,200	
91,200	1,356,894	1,488,099	08:59:00	00:07:59	10%	*	4,283,797	05:03:00
161,600		2,632,201		00:15:50		*		

* - Out of memory for NEDP2, best results presented, if available.

6 Conclusions

1. A new multi-objective model for network design, which is evacuation oriented was developed. This model enables the policy makers to analyze different network designs in terms of construction cost and evacuation time. Such a model is especially relevant for facilities sensitive to mass evacuation (military depot, civilian population close to high-risk areas, stadiums, etc.).
2. As the model is NP-complete, an efficient heuristic was developed, which is both fast, easy to construct, and performs well, when compared to the optimal algorithm.
3. The use of the well known minimum-cost problem as the basis for the heuristic, assures an effective implementation for very large problems, as many efficient variations of the minimum-cost problem exist.
4. Further research of existing networks can be carried out to investigate network improvement for different evacuation scenarios.

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