Worst-Case Analysis for the Split Delivery Vehicle Routing Problem with Minimum Delivery Amounts

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Abstract

In the vehicle routing problem (VRP), a fleet of vehicles must service the demands of customers. A vehicle begins and ends its route at the same depot and the sum of the demands of the customers on a route cannot exceed a vehicle's capacity. A

customer must have all of its demand delivered at one time by a single vehicle. The objective is to minimize the total distance traveled by the fleet.

In the split delivery vehicle routing problem (SDVRP), more than one vehicle is allowed to service a customer, so that a customer's demand can be split among several vehicles on different routes. The objective in the SDVRP is to minimize the total distance traveled by the fleet, while satisfying the demand of each customer.

Archetti, Savelsbergh, and Speranza [1] provide a worst-case analysis of the SDVRP. They show that, by allowing split deliveries, travel distance can be reduced by at most 50%, and this is a tight bound. That is, they showed $\frac{Z(VRP)}{Z(SDVRP)} \leq 2$, where $Z(VRP)$ is the distance of an optimal VRP solution to an instance (no splits allowed), and $Z(SDVRP)$ is the distance of an optimal SDVRP solution to the same instance (splits allowed). Furthermore, there exist instances for which $\frac{Z(VRP)}{Z(SDVRP)}$ is arbitrarily close to 2.

Recently, Gulczynski, Golden and Wasil [2] consider the split delivery vehicle routing problem with minimum delivery amounts (SDVRP-MDA). In the SDVRP-MDA, split deliveries are allowed only if at least a minimum fraction of a customer's demand is delivered by each vehicle visiting the customer. For example, if $p = .2$ is the minimum delivery fraction, and $D_i = 10$ is the demand of customer i, then each vehicle visiting customer i must deliver at least $pD_i = 2$ units. A split delivery of, say, 1 unit from one vehicle and 9 units from another vehicle would not be allowed, since 1 is less than the minimum delivery amount. The objective of the SDVRP-MDA is the same as for the SDVRP: minimize the total distance traveled by the fleet, while satisfying the demand of each customer.

Notice, in the SDVRP-MDA the minimum delivery fraction p must be at least 0 and at most 1. When $p = 0$ the SDVRP-MDA reduces to the SDVRP, and when $p > .5$ the SDVRP-MDA reduces to the VRP. Thus, in this paper, we focus on the minimum delivery fractions p for which $0 < p \leq .5$.

Since the SDVRP-MDA is a generalization of the SDVRP, it is natural to wonder what properties of the SDVRP extend to the SDVRP-MDA. Gulczynski, Golden, and Wasil [2] briefly consider properties of the SDVRP-MDA in their paper. One of their results gives bounds for a worst-case SDVRP-MDA scenario. Let $Z(VRP)$ be the distance of an optimal VRP solution to an instance, and let $Z_p(MDA)$ be the distance of an optimal SDVRP-MDA solution to the same instance with minimum delivery fraction p. Further, let $M(p)$ be the least upper bound of $\frac{Z(VRP)}{Z_p(MDA)}$. They showed that $2 - p \leq M(p) \leq 2$.

Notice that the result of Gulczynski, Golden, and Wasil [2] leaves open the possibility that the bound of $\frac{Z(VRP)}{Z_p(MDA)}$ depends on p. Surprisingly, this turns out to *not* be the case for almost all p. In this paper, we prove that for p, $0 < p < .5$, $M(p) = 2$. That is, the worst-case bound for the SDVRP-MDA is independent of p, and it is the same as that for the SDVRP. When $p = .5$ this result does not hold. In the interesting special case of $p = .5$, we have that $M(p) = 1.5$.

References

[1] C. Archetti, M. Savelsbergh, and M. Speranza, *Worst-case analysis for split delivery vehicle routing problems*, Transportation Science, 40 (2006), pp 226-234.

[2] D. Gulczynski, B. Golden, and E. Wasil, *The split delivery vehicle routing problem with minimum delivery amounts*, Transportation Research Part E, 46 (2010), pp 612-626.