Risk Aversion and Retail Electricity Markets

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Abstract

In 2009 Prof. Frank Wolak carried out a review of the New Zealand electricity market. In this report he noted that there is a lack of competition in the NZEM, particularly during dry years and suggested that competition could be improved by reallocating generation assets amongst the market participants. We have designed an equilibrium model that encompasses both the retail and wholesale electricity sectors. In the future, this model will enable us to gain insights into the effects of asset reallocation.

Key words: electricity markets, game theory, risk aversion, equilibrium.

1 Introduction

Over the past two years, there has been much discussion regarding the competitiveness of the New Zealand electricity market (NZEM). In May 2009, the Commerce Commission released a report by Stanford University's Professor Frank Wolak [9]. This report highlighted that firms in the wholesale market possess *transient* market power. That is, firms occasionally have the ability (and the incentive) to increase spot prices significantly above the underlying long-run marginal costs of their power plants. This generally occurs during times when hydro reservoir levels are low (meaning that the opportunity cost of water is high). The Wolak report also raised a concern that only one firm in the NZEM owns significant generation in both islands, which can lead to limited competition at times when the HVDC link is constrained. Wolak suggested that reallocating generation assets amongst the state-owned firms could reduce the ability of firms to exercise this market power, in the wholesale market.

Later in 2009, a discussion document from the Electricity Technical Advisory Group [2], agreed with Wolak's recommendation of an asset reallocation and suggested three alternatives. This report stated that this reallocation should improve competition in the retail market, by incentivising *gentailers* (firms with both generation and retail arms) to enter into new markets; thereby driving prices down for consumers.

In this paper, we investigate retail competition over a *competitive* wholesale market, and attempt to understand how risk aversion on the part of gentailers may affect their retail pricing decisions and their incentives to enter new markets. This paper is laid out as follows: in the next section, we outline how we model and measure risk on behalf of retailers; we then present our model of retail markets; next, we discuss the clearing of the wholesale market. A simple example is constructed to illustrate the impact of risk-aversion on retail pricing and finally a general model formulation is presented that can, in the future, be used to examine the impact of asset reallocation.

1.1 Risk

When companies are making decisions regarding investment or entering contracts they must not solely consider the expected benefit from the decisions, they must also take into account the consequences if the return on the investment is lower than expected – this is known as risk-aversion. Companies are responsible to shareholders over both the short and long term, so they need to make decisions that seek to minimize their risk, while maximizing expected profit. Mean-risk optimization is commonly used in portfolio optimization; this approach involves solving a bi-objective optimization problem and typically results in finding a set of Pareto optimal solutions, known as the *efficient frontier*. In such models risk is typically measured by the variance of the return or by using the downside-risk, as introduced by Markowitz [7].

Another common way of incorporating a risk-attitude into optimization problems is to use a utility function. If a business is risk-averse, a concave, increasing utility function will reflect this. That business can then maximize its expected utility (as a function of profit) rather than its expected profit; the optimal solution to this will have a lower (or equal) expected return that if the firm had been profit maximizing, but the risk associated with this decision will be less. However, determining a precise utility function is difficult.

In order to formalize the concept of risk, *coherent* measures of risk were introduced by Artzner et al. in [1]. Any coherent risk measure must comply with the following four axioms: subadditivity; translation invariance; positive homogeneity; and monotonicity. Risk measures complying with these axioms exhibit key properties that are valuable for a risk-averse agent. For example, subadditivity ensures that there is a risk-pooling effect: the sum of risks is greater than or equal to the risk of the sum.

Conditional value at risk (CVaR)[1] is a coherent risk measure. The CVaR at level β of some random profit π is simply the expected loss if one's interest were restricted to the lowest $100\beta\%$ of returns. If profits π are continuously distributed with some distribution function $F(\pi)$ and associated probability density function $f(\pi)$ then $\text{CVaR}_\beta(\pi)$ can be written as:

$$\text{CVaR}_\beta\left(\rho\right) = -\frac{1}{\beta}\int_{-\infty}^{F^{-1}(\beta)} \pi f\left(\pi\right) d\pi.$$

Initially this measure of risk may seem impractical for optimization problems. The above description requires the order of the profits to be known a priori, when in fact, in many circumstances, the ordering of outcomes often will depend on the decision variables being optimized over. Fortunately, however, Shapiro et al. [8] present an alternate

[1]This is all known as average value at risk, or expected shortfall.

formulation in which the bottom $100\beta\%$ of outcomes can be computed through a linear program. Moreover, for profit functions that are concave in the decision variables, we can maximize a weighted combination of expected profit and risk. In the formulation below, α is a parameter between 0 and 1 and changes the weightings on risk and return[2]:

$$\max \qquad E_\pi [\pi] - \frac{\alpha}{\beta} E_\pi [\max \{(1-\beta)(\eta - \pi), \beta(\pi - \eta)\}]. \qquad (1)$$

In this paper, we use the above mean-risk formulation to create a model of an electricity retail market with risk-averse firms. We will initially compare the behaviour of firms, competing in the retail power sector, who are risk-neutral with those that are risk-averse. A risk-neutral firm is not concerned about the need to hedge against possible negative outcomes; they are solely interested in maximizing their expected profit. Conversely, risk-averse firms are concerned with mitigating the adverse affects of negative outcomes (such as, in electricity markets, plant outages or high fuel costs) by changing their exposure to the spot market by altering their expected retail positions.

1.2 Retail Competition

Modelling of competition in the retail sector of electricity markets requires an understanding of consumer behaviour. In the literature, many models of consumer behaviour have been analysed (see, e.g., [6, 5]). These models are typically based on price competition, whereby firms submit their prices to the market and consumers choose a generator which offers the best deal. Under the assumption of Bertrand competition ([4]), this would mean that all consumers will go to the firm offering the lowest price, however, this is not empirically observed. Hotelling models ([5]) build upon this basic principle of price competition, but here the consumers may have an initial preference to a firm or there may be additional cost unique to each consumer for each firm.[3] This creates some elasticity in the market, allowing firms to set their price above that of competing firms, but still retain some customers.

Another branch of work concerned with consumer behaviour are consumer switching models. Here consumers have costs of switching or searching out firms. These models may allow incumbent firms to overcharge consumers, due to the consumers' reluctance to seek out better deals; see, e.g, [6].

2 Model

In this section we introduce the model that we will use. It consists of three stages, in the first stage firms decide whether or not they wish to participate in the retail market. In the following stage, firms participating in the retail market choose retail prices which they set for an extended period (for example, one year). In the third stage of the model, the uncertain wholesale market clears. We will discuss the final two stages in more detail below, starting with the wholesale market.

[2]See the Appendix, or [8] for a derivation of this expression.

[3]For example, the distance a consumer must travel may influence a consumer to purchase from the more expensive retailer.

2.1 Wholesale Market

In this work, for simplicity, we assume that the wholesale market is *perfectly competitive*. This is the simplest assumption about the behaviour of the firms, since the offers are set to the plants' marginal costs. These offers are received by the system operator and an optimization problem (the *economic dispatch problem*) is solved to minimize the cost of satisfying the demand, yielding the optimal generation, power flows and nodal prices. These nodal prices are defined to be the cost of an additional unit of power at a node.

Although, at the time that the system operator solves the dispatch problem there is no uncertainty, when firms are setting their retail prices there may be uncertainty surrounding the costs of the firms (water value or gas costs) or capacities of plants and lines (due to outages).

2.2 Retail Competition

We use a differentiated products formulation to model the demand each firm receives as a function of their price; here we assume that the products that the firms offer are partial strategic substitutes. Thus if each firm i chooses a price p_i, we can compute their demand from the following function:

$$x_i = X_i + \sum_{j \neq i} f_{ij}\left(p_i, p_j\right),$$

where X_i is the default load if all firms charge the same price and $f_{ij}(p_i, p_j)$ is any increasing function satisfying the following conditions[4]:

$$\begin{aligned} \frac{\partial f_{ij}}{\partial p_i} &\leq 0, \\ \frac{\partial f_{ij}}{\partial p_j} &\geq 0, \\ \left.\frac{\partial f_{ij}}{\partial p_i}\right|_{p_i=p_i^*} &\leq \left.\frac{\partial f_{ji}}{\partial p_i}\right|_{p_i=p_i^*}. \end{aligned}$$

If we define $f_{ij}\left(p_j - p_i\right) = b\left(p_j - p_i\right)$ and $X_i = \frac{X}{n}$ then we have

$$x_i = \frac{X}{n} - b\left(n-1\right)p_i + b\sum_{j \neq i} p_j.$$

Note that in the above model the overall demand is inelastic; however, for the individual firms, demand is lost or gained based on the cross-elasticity constraint b and the price differences between firms. For ease of computation, we will restrict our interest to the linear demand model, above, for the remainder of this paper.

[4]These conditions ensure that as the price of a firm increases the demand for that firm decreases and the demand of other firms increases. Moreover, the number of customers leaving one firm must be less than or equal to the number arriving at another.

2.3 Risk-neutral firms

If firms are risk neutral, this means they aim to maximize their expected profit. We can observe this in the single node case below. The (expected) profit function for firm i is given by

$$\begin{aligned}\pi_i &= E_\omega\left[(p_i - c)\left(\frac{X}{n} - b(n-1)p_i + b\sum_{j\neq i} p_j\right) + P_i\right] \\ &= E_\omega\left[(p_i - c)\left(\frac{X}{n} - b(n-1)p_i + b\sum_{j\neq i} p_j\right)\right] + E_\omega[P_i] \\ &= (p_i - E_\omega[c])\left(b\sum_{j\neq i} p_j - b(n-1)p_i\right) + p_i\frac{E_\omega[X]}{n} - E_\omega\left[\frac{cX}{n}\right] + E_\omega[P_i],\end{aligned}$$

where P_i is firm i's profit from the wholesale market.

Now we wish to compute the equilibrium to the retail game; this is a point where no firm has a strategy that can unilaterally improve its expected profit. Note that each player has a smooth, concave profit function, thus we can find the maximum profit from the first order condition; differentiating the expected profit function with respect to p_i gives

$$\frac{\partial \pi_i}{\partial p_i} = \frac{E_\omega[X]}{n} - 2(n-1)bp_i + b\sum_{j\neq i} p_j + (n-1)bE_\omega[c].$$

Now we solve for price p_i^* yielding maximum profit:

$$\left.\frac{\partial \pi_i}{\partial p_i}\right|_{p_i = p_i^*} = 0 \Rightarrow p_i^* = \frac{\frac{E_\omega[X]}{n} + b\sum_{j\neq i} p_j + (n-1)bE_\omega[c]}{2(n-1)b} \qquad (2)$$

We sum the above equation for all i, yielding

$$\sum_i p_i^* = \frac{E_\omega[X] + \sum_i p_i + n(n-1)bE_\omega[c]}{2(n-1)b},$$

which can be rearranged to give

$$\sum_i p_i^* = \frac{E_\omega[X]}{(n-1)b} + nE_\omega[c].$$

Finally we substitute the above expression into equation (2) to find the equilibrium prices:

$$p_i^e = \frac{E_\omega[X]}{n(n-1)b} + E_\omega[c],$$

For scenario ω the retail demand that the firm gets is:

$$x_i^* = \frac{X_\omega}{n}.$$

In other words, we find that the firms all choose identical prices for their retail sales, and each firm gets an equal market share. This is because the profit from the wholesale market is not affected by the decisions made in the retail market, and is merely a (uncertain) constant term in the profit function. In a risk neutral setting, the expected value of this wholesale profit can be separated from the retail profit, so it plays no part in the optimization problem. On the surface this may seem peculiar, but optimizing expected profits cannot distinguish between opportunity costs and actual costs, so all firms have the same incentives. We illustrate this by way of an example in the next section.

2.3.1 Single-node Example

Consider a situation with one node having a totally inelastic demand of 150MW (all retail). In the market there are two firms, each with a retail base: firm A owns a hydro plant with capacity 100MW and cost \$$h$/MWh and firm B owns a thermal plant with capacity 100MW and cost \$50/MWh. The hydro costs are uncertain when the retail prices are chosen, however, it is common knowledge that the hydro costs, h are distributed uniformly from 0 to 100.

First, we solve for the optimal dispatch as a function of h. This gives the following dispatch quantities, q_A, q_B for each plant and clearing price, c:

$$\begin{aligned} q_A &= \begin{cases} 100, & h \leq 50, \\ 50, & h > 50, \end{cases} \\ q_B &= \begin{cases} 50, & h \leq 50, \\ 100, & h > 50, \end{cases} \\ c &= \begin{cases} 50, & h \leq 50, \\ h, & h > 50. \end{cases} \end{aligned}$$

Now we can compute the expected profits of the firms as below:

$$\begin{aligned} E[\pi_A] &= E[q_A \times (c-h) + d_A (p_A - c)] \\ &= \tfrac{1}{100} \int_0^{50} 100(50-h)\,dh + d_A \int_0^{50} (p_A - 50)\,dh + d_A \int_{50}^{100} (p_A - h)\,dh \\ &= 1250 + d_A (p_A - 62.5). \end{aligned}$$

Substituting in d_A (with $b = 1$) and differentiating with respect to p_A gives:

$$\frac{\partial E[\pi_A]}{\partial p_A} = 75 - 2p_A + p_B + 62.5.$$

Similarly for firm B we find

$$\frac{\partial E[\pi_B]}{\partial p_B} = 75 - 2p_B + p_A + 62.5.$$

Assuming an interior solution, these first order conditions yield the following equilibrium prices:

$$p_A^e = p_B^e = 137.5.$$

2.4 Risk-averse firms

In a risk-averse setting, however, this symmetry is lost. This is because the outcome of the wholesale market cannot be merely treated as a constant in the risk-adjusted profit maximization problem:

$$\rho_{\alpha,\beta}\left[\pi_i\right] = (1-\alpha)E\left[\pi_i\right] - \alpha\mathrm{CVaR}_\beta\left[\pi_i\right].$$

In the next example, we will demonstrate how risk aversion alters the optimal decisions of the firms.

2.4.1 Single-node Example

For this example, we will define the CVaR risk level, β=0.1, examine the affect of varying α on the behaviour of the firms. In order to compute the conditional value at risk, we need to rank the profit for each firm as a function of h for each retail price is may choose. The revenue for a given h and p_A for firm A is:

$$\pi_A = \begin{cases} (p_A - 50)(75 - p_A + p_B) + 100(50 - h), & h \le 50, \\ (p_A - h)(75 - p_A + p_B), & h > 50. \end{cases}$$

Note that, from the above equation, we can see that so long as the the retail demand is positive the profit of firm A is decreasing in h. The revenue for a given h and p_B for firm B is:

$$\pi_B = \begin{cases} (p_B - 50)(75 - p_B + p_A), & h \le 50, \\ (p_B - h)(75 - p_B + p_A) + 100(h - 50), & h > 50. \end{cases}$$

From above we can see that firm B's profit is constant over $h \in [0, 50]$, however for $h > 50$ we have:

$$\frac{\partial \pi_B}{\partial h} = 25 + p_B - p_A,$$

hence if $p_A - p_B \ge 25$ then the profit is decreasing in h, whereas if $p_A - p_B \le 25$ the profit is increasing in h; it can be shown that the former case is not possible at equilibrium, hence we can write the risk-adjusted profit functions for both firms:

$$\begin{aligned} \rho_{\alpha_A,0.1}\left[\pi_A\right] &= \frac{1-\alpha_A}{100}\int_0^{100}\pi_A(h)\,dh + \frac{\alpha_A}{10}\int_{90}^{100}\pi_A(h)\,dh \\ &= (1-\alpha_A)\left[(p_A - 62.5)(75 + p_B - p_A) + 1250\right] + \alpha_A\left[(p_A - 95)(75 + p_B - p_A)\right], \end{aligned}$$

$$\begin{aligned} \rho_{\alpha_B,0.1}\left[\pi_B\right] &= \frac{1-\alpha_B}{100}\int_0^{100}\pi_B(h)\,dh + \frac{\alpha_B}{10}\int_0^{10}\pi_B(h)\,dh \\ &= (1-\alpha_B)\left[(p_B - 62.5)(75 + p_A - p_B) + 1250\right] + \alpha_B\left[(p_B - 5)(75 + p_A - p_B)\right]. \end{aligned}$$

We can then solve for the equilibrium, from first order conditions, giving:

$$p_A^e = \frac{5}{6}\left(165 + 26\alpha_A - 23\alpha_B\right), \quad p_B^e = \frac{5}{6}\left(165 + 13\alpha_A - 46\alpha_B\right).$$

Note that if we set $\alpha_A = \alpha_B = 0$ then we recover the risk-neutral equilibrium. Moreover, as firm A's risk-aversion weighting is increased the equilibrium prices of both players increase; whereas as firm B's risk-aversion weighting is increased the equilibrium prices of both players decrease. This result is due to the wholesale market positions of the firms. Firm A, who owns hydro generation receives the least profit when hydro costs are high, thus a risk-averse firm emphasizes these occurrences and when optimizing in the retail market sees a higher risk-adjusted 'average' spot price. Conversely, firm B, who owns thermal generation receives the least profit when hydro costs are low, thus when optimizing its retail price sees a lower risk-adjusted 'average' spot price. Furthermore, the effects of competition mean that the risk-aversion of one firm will affect the other's optimal pricing strategy.

This example has illustrated how risk-aversion can affect the retail equilibrium prices. In the next section we formulate the retail problem more generally and discuss the solution techniques.

2.4.2 General formulation

Now let us define the price-competition amongst firms $j \in \mathcal{F}$. The firms are able to compete for retail customers at any node $i \in \mathcal{N}$ in the network. However, there is uncertainty about future demand levels and nodal prices. The different scenarios are indexed by $s \in \mathcal{S}$.

Each firm j solves the following optimization problem to optimize its risk-adjusted profit function.

Parameters:

- c_{si} is the wholesale price in scenario s at node i;
- A_{ij} is a boolean matrix specifying whether a firm i competes in the retail market at node i;
- X_{sij} is the retail demand for firm j at node i in scenario s when all retailers offer the same price.
- p_{ik} is the retail price offered by firm $k(\neq j)$ at node i.
- pr is the probability of each scenario (assuming all scenarios are equally likely).

Variables:

- η_j is the β-quantile profit of firm j;
- v_{sj} is the positive deviation from the β-quantile profit of firm j;
- w_{sj} is the negative deviation from the β-quantile profit of firm j;
- d_{sij} is the realised retail demand for firm j at node i in scenario s;
- Z_{sj} is the profit for firm j in scenario s;
- p_{ij} is the retail price offered by firm j at node i.

Formulation:

$$
\begin{array}{lll}
\max & \sum\limits_s pr \times Z_{sj} - \frac{1}{\beta} \sum\limits_s pr \left[(1-\beta)\, w_{sj} + \beta v_{sj} \right] & \\
\text{s.t.} & Z_{sj} = \eta_j + v_{sj} - w_{sj} & [\lambda_{sj}] \\
& Z_{sj} = \sum\limits_i A_{ij} \left(p_{ij} - c_{si} \right) d_{sij} + R_{sj} & [\mu_{sj}] \\
& d_{sij} = X_{sij} + b \sum\limits_k A_{ik} \left(p_{ik} - p_{ij} \right) & [\nu_{sij}] \\
& w_{sj}, v_{sj} \geq 0. &
\end{array}
$$

The above problem is convex in the decisions variables p_{ij}, $\forall i \in \mathcal{N}$, therefore KKT conditions are equivalent. Below we hold the KKT conditions of all firms $j \in \mathcal{F}$ simultaneously.

$$
\begin{array}{ll}
Z_{sj} = \eta_j + v_{sj} - w_{sj} & \forall s \in \mathcal{S},\ \forall j \in \mathcal{F}, \\
Z_{sj} = \sum\limits_i A_{ij} \left(p_{ij} - c_{si} \right) d_{sij} + R_j & \forall s \in \mathcal{S},\ \forall j \in \mathcal{F}, \\
d_{sij} = X_{sij} + b \sum\limits_k A_{ij} \left(p_{ik} - p_{ij} \right) & \forall s \in \mathcal{S},\ \forall i \in \mathcal{N},\ \forall j \in \mathcal{F}, \\
-pr - \lambda_{sj} - \mu_{sj} = 0 & \forall s \in \mathcal{S},\ \forall j \in \mathcal{F}, \\
A_{ij} \left(p_{ij} - c_{si} \right) \mu_{sj} - \nu_{sij} = 0 & \forall s \in \mathcal{S},\ \forall i \in \mathcal{N},\ \forall j \in \mathcal{F}, \\
\sum\limits_s \left[A_{ij} d_{sij} \mu_{sj} + b \sum\limits_k A_{ik} \left(-\nu_{sij} \right) + b \nu_{sij} \right] & \forall i \in \mathcal{N},\ \forall j \in \mathcal{F}, \\
\sum\limits_s \lambda_{sj} = 0 & \forall j \in \mathcal{F}, \\
\lambda_{sj} + pr \geq 0 & \forall s \in \mathcal{S},\ \forall j \in \mathcal{F}, \\
-\lambda_{sj} + \frac{1-\beta}{\beta} pr \geq 0 & \forall s \in \mathcal{S},\ \forall j \in \mathcal{F}.
\end{array}
$$

This system of equations is known as a mixed complementarity problem; this type of problem can be solved in `GAMS` using the `PATH` solver [3]. We will not present any models of the New Zealand system here; they will be available in a future paper.

3 Conclusions

Motivated by Prof. Frank Wolak's report on the NZEM and the Ministerial Review into the electricity industry, we have constructed a model which encompasses both the retail and wholesale markets. We first observe that with risk-neutral firms and a competitive wholesale market, retailers will reach a symmetric equilibrium. We then examine how their behaviour may change using a simple one-node example with two firms. We find that risk-aversion does not always mean that a premium is passed onto consumers, in fact, retail prices may drop as a firm becomes more risk-averse (this effect is dependent upon the particular circumstances of the retailers).

We finish this paper with a model formulation for computing an retail market equilibrium with arbitrary sets of firms, nodes and wholesale market scenarios.

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Appendix

Conditional value at risk as a mean–risk measure

The following function of the random variable Z gives the expected weighted deviation from any given quantile β

$$r_\beta [Z] = \min_\eta \{\varphi(\eta) := E[\max\{(1-\beta)(\eta - Z), \beta(Z-\eta)\}]\}.$$

To see that the optimal η does in fact correspond to the β-quantile, we first take left and right derivatives of the above function with respect to η:

$$\begin{aligned}\varphi'(\eta)_+ &= \Pr[Z \leq \eta] \times (1-\beta) - \Pr[Z > \eta] \times \beta \geq 0,\\ \varphi'(\eta)_- &= \Pr[Z < \eta] \times (1-\beta) - \Pr[Z \geq \eta] \times \beta \leq 0.\end{aligned}$$

Observe that at the optimal η the left derivative must be non-increasing and the right derivative must be non-decreasing.[5] We can then rearrange the above inequalities to find:

$$\Pr[Z < \eta] \leq \beta \leq \Pr[Z \leq \eta],$$

which confirms that the optimal η is the β-quantile.

Hence if $F(\alpha)$ is the cumulative distribution function and $p(\alpha)$ is the probably distribution function corresponding to the random variable $Z(\alpha)$, then

$$\begin{aligned} r_\beta[Z] &= \int_0^\beta p(\alpha)(1-\beta)\left(F^{-1}(\beta) - Z(\alpha)\right) d\alpha + \int_\beta^1 p(\alpha)\beta\left(Z(\alpha) - F^{-1}(\beta)\right) d\alpha \\ &= \int_0^\beta p(\alpha)(1-\beta)F^{-1}(\beta)\,d\alpha - \int_0^\beta Z(\alpha)p(\alpha)(1-\beta)\,d\alpha + \int_\beta^1 Z(\alpha)p(\alpha)\beta d\alpha - \int_\beta^1 F^{-1}(\beta)p(\alpha)\beta d\alpha \\ &= \int_\beta^1 Z(\alpha)p(\alpha)\beta d\alpha - \int_0^\beta Z(\alpha)p(\alpha)(1-\beta)\,d\alpha \\ &= (1-\beta)\beta\left(E_{Z\geq Z(\beta)}[Z] - E_{Z\leq Z(\beta)}[Z]\right).\end{aligned}$$

Moreover, note that the expectation of Z can be written as:

$$E[Z] = \beta E_{Z\leq Z(\beta)}[Z] + (1-\beta)E_{Z\geq Z(\beta)}[Z].$$

Finally, we find that

$$\begin{aligned} E[Z] - \frac{1}{\beta} r_\beta[Z] &= \beta E_{Z\leq Z(\beta)}[Z] + (1-\beta)E_{Z\geq Z(\beta)}[Z] - (1-\beta)\left(E_{Z\geq Z(\beta)}[Z] - E_{Z\leq Z(\beta)}[Z]\right) \\ &= \beta E_{Z\leq Z(\beta)}[Z] + (1-\beta)E_{Z\leq Z(\beta)}[Z] \\ &= E_{Z\leq Z(\beta)}[Z] = \mathrm{CVaR}_\beta(Z).\end{aligned}$$

This can be incorporated into a mean-risk optimization problem with a parameter $\alpha \in [0,1]$ controlling the weightings on risk versus mean return.

$$\begin{aligned}(1-\alpha)E[Z] - \alpha \mathrm{CVaR}_\beta(Z) &= (1-\alpha)E[Z] + \alpha\left(E[Z] - \frac{1}{\beta}r_\beta[Z]\right) \\ &= E[Z] - \frac{\alpha}{\beta} r_\beta[Z].\end{aligned}$$

[5] Of course this assumes that φ is continuous in η, which can be easily verified.

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