A Decision Support Tool for Equipment Replacement in Forestry Harvesting Operations

Matthew Baxter
Department of Mathematics and Statistics
University of Melbourne
Australia
m.baxter3@pgrad.unimelb.edu.au

Mark Brown
Department of Forestry and Ecosystem Science
University of Melbourne
Australia
mwbrown@unimelb.edu.au

Heng-Soon Gan
Department of Mathematics and Statistics
University of Melbourne
Australia
hsg@unimelb.edu.au

Abstract

While managing forests for diverse and conflicting outcomes requires great care and patience over many years, the largest expense associated with producing timber products is the cost of harvesting. The contractors responsible for this task must invest in several types of expensive harvesting machinery with limited lifespans, with the goal of minimising their operating cost per tonne including machine ownership costs. Much work has been done on calculating the approximate total cost of owning a machine, given its expected lifespan and other parameters. While extremely useful, these calculations neglect several effects that can only be seen by considering a machine as a part of the larger operation. In this paper, we describe several mixed-integer linear programming models, with varied levels of complexity, developed to make decisions regarding equipment replacement. We further describe a prototype decision support tool to be used by machine owners.

Key words: forestry operations, equipment replacement, mixed integer linear programming, decision support tool

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1 Introduction

Much work has been done on calculating the approximate total cost of owning a machine, given it's expected lifespan and other parameters. While extremely useful, these calculations neglect several effects that can only be seen by considering a machine as a part of the larger operation. The operation can only be as productive as its least productive class of machines, and this should be considering during purchasing decisions. The lifespan of a machine impacts its total cost of ownership considerably. Currently, the manufacturer's recommended lifespan is used in calculations, but it may be more cost effective in the long term to replace the machine earlier or to keep it running well past its recommended lifespan. This project will consider the cost of machines as a long-term planning problem.

Contractors are engaged by forest owners/managers, typically on a five-year contract, to harvest products from specified strands of trees. These contactors operate several classes of machines, each class having its own role, with the set-up of the operation determining which classes are required. For example, an operation producing woodchips may use Feller-Bunchers to fell and bunch trees, a skidder to move them to roadside then a Chipper to create woodchips ready to be trucked to the client.

With the exception of trucks, a contractor is likely to have no more than a few of each class of machine. Additionally, the cost of purchasing and maintaining these machines is a large proportion of a contractor's costs. For these reasons, deciding when to replace a machine and selecting a model to replace it with is a highly important problem.

The paper will consider the Equipment Replacement Problem in forestry harvesting operations. Typically, we aim to answer questions including "Which setup to use?", "What models to buy? (size, power, manufacturer...)", "When to buy and salvage machines?", "How to distribute the workload when the machines have greater capacity than required?" and "How much should the Forest Owner be charged for harvesting?".

If the equipment is always replaced with identical equipment (with identical costs over its age) and the time horizon is infinite (or only the first replacement is considered), then the optimal solution is to always replace the equipment at the end of its economic lifetime. This is the lifetime which minimises its average cost per unit of time (including purchase cost). This approach can be adjusted to account for technological improvement (Rogers and Hartman 2005) and/or inflation (a linear relationship between cost and time).

With a finite time horizon and discrete time periods, a dynamic programming algorithm exists which finds optimal solutions for the single machine case. (Waddell 1983) describes such an approach. This algorithm runs quickly enough to be run for large time horizons, and does not require the assumption that costs are linear with regard to time. However the algorithm is still limited in that it is designed for the single machine problem, and expanding it to multiple machines requires the assumption that there is no interdependence between the decisions.

The common approaches to equipment replacement used in Forestry ignore any and all interdependency between replacement decisions and do not attempt to calculate the economic lifetime of equipment. See for example (Brinker et al. 2002; Murphy and Associates 2008; Bilek 2007; Bilek 2009).

(Burt et al. 2010; Burt 2008)'s work, which is applied to the mining industry, is by far the closest analogue to the problem faced by the forestry industry. It involves a Multi–Period Equipment Selection problem (with heterogeneous fleets), which is largely equivalent to the type of Equipment Replacement problem which is described here. A brief section of this paper will be dedicated to compare and contrast (Burt et al. 2010; Burt 2008)'s model with our model.

The plan of this paper is as follows. We will start by describing various mathematical formulations in §2. The decision support tool is described in §3, and we conclude in §4.

2 Mathematical Formulations

The purpose of this model is to create a strategic plan for buying machines and distributing the work between them in order to maximise profits. The model is able to select between different possible contracts and configurations in order to meet these goals. However it is expected that in most applications the user will already have a contract in mind (and can make an educated guess regarding future contracts) and will be locked into a particular configuration. The contract income can be ignored by the model in those cases.

Each year is modelled as a discrete period, and the model has a finite time horizon of T years. The following concepts will be used throughout the model:

Contract: A contract specifies an amount of work to be done for a given price. For each possible contract this model requires knowledge of the amount of work required to fulfill the contract and the expected profit before machine costs. Details given in the section on parameters.

Time block: This is a set of consecutive time periods, blocked to represent the effective duration of a contract. This will typically be a five-time-period block.

Machine group: This is also known as a machine class or machine type. For example; feller-buncher, processor, forwarder or loader. There exist different makes of machine in each machine group.

Harvest system: This is the set of machine groups required to work together to convert "raw materials" into "finished products". Only one harvest system is chosen for a given contract over the effective time block.

Key to the understanding of the problem is the knowledge that forestry contractors cannot simply produce as much product as they wish in order to increase profits. They have a maximum level of production specified by the contract. Most forestry contracts allow for less product to be produced, so it could on occasion be more profitable not to reach the contract figure in a given year. However this is considered poor practice and may jeopardise the contractors reputation and future contracts. Therefore this model considers the contracted amount of work to be a hard constraint, and when used to choose between models considers their net profit before machine costs (ignoring the price per tonne).

The contract will generally give this constraint in tonnes per year whereas this model needs to know how much work is required from each machine group (given a particular harvest system) for every year. The relationship between the two is not straightforward; the amount of work per tonne varies immensely, and depends on different factors for different machine groups. Tools such as ALPACA(Murphy and Associates 2008) could be useful in calculating one from the other and experienced contractors are likely to be able to give good estimates.

Machines also vary in performance, will break down more often as they get older and may suffer additional performance penalties. This is accounted for in our model using piecewise linear functions to model the actual productive hours.

The index sets used by the model are:

$\mathbf{T}$	:	the set of time periods $\{1, \ldots, T\}$ (one time period equals one year)
$\mathbf{B}$	:	set of time blocks (one time block equals five consecutive time periods)
$\mathbf{T}_b \subset \mathbf{T}$	:	set of time periods in block $b \in \mathbf{B}$ (mutually exclusive)
$\mathbf{G}$	:	set of machine groups (classes or types)
$\mathbf{M}$	:	set of machines
$\mathbf{M}_g \subset \mathbf{M}$	:	set of machines in the machine group $g \in \mathbf{G}$
$\mathbf{C}$	:	set of contracts
$\mathbf{C}_b \subset \mathbf{C}$	:	set of contracts in time block $b \in \mathbf{B}$
$\mathbf{O}_c$	:	set of harvest systems for contract $c \in \mathbf{C}$

Parameters:

T	:	planning horizon
H	:	number of working hours in a year.
C_{mij}	:	unit cost of purchasing machine $m \in \mathbf{M}$ at the START of, period $i \in \mathbf{T}$ and salvaging it at the END of period $j \in \mathbf{T}$ (include interest, inflation, depreciation rates and insurance costs)
R_c	:	the net profit expected on contract $c \in \mathbf{C}$ before machine costs
W_{gkt}	:	the amount of productivity required from machine group $g \in \mathbf{G}$ in time period $t \in \mathbf{T}$ if the harvest system $k \in \mathbf{O}_c$ is chosen for contract $c \in \mathbf{C}$

Decision variables:

x_{kb}	=	1 if harvest system $k \in \mathbf{O}_c$ is chosen for contract $c \in \mathbf{C}_b$ in time block $b \in \mathbf{B}$, 0 otherwise
n_{mij}	=	the number of machines of type $m \in \mathbf{M}$ are purchased at the START of period $i \in \mathbf{T}$, and salvaged at the END of period $j \in \mathbf{T}$ (note: the (i, j) combinations may be restricted by the machine replacement policy;)
w_{mijt}	=	the amount of scheduled hours done by one unit of machine of type m, which were bought at time i and sold at j, during time period t. (We assume here that all machines of type m bought at i and salvaged at the end of j is given the same workload, thus giving a total workload of $n_{mij}w_{mijt}$ for this set of machines for time period t.)

Let $f_m(\theta)$ be a piecewise linear function that gives the total amount of work (in idealised productive hours) that has been done by a machine of age θ (in scheduled hours) and $g_{mt}(\theta)$ be a piecewise linear function that gives the maintenance costs incurred by that machine during time period t. The function $g_{mt}(\theta)$ is in dollars, defined for each time period t to support adjustments for the time value of money. We assume, without loss of generality, that the linear intervals of f_m and g_{mt} (for all t) are the same.

We designate the following mathematical program as model FHER. Expressing in general terms for functions f_m and g_{mt}, model FHER aims to maximise

$$\sum_{b\in\mathbf{B}}\sum_{c\in\mathbf{C}_b}\sum_{k\in\mathbf{O_c}} R_c x_{kb} - \sum_{m\in\mathbf{M}}\sum_{i,j\in\mathbf{T}} C_{mij} n_{mij}$$
$$- \sum_{m\in\mathbf{M}}\sum_{i,j\in\mathbf{T}}\sum_{i\le t\le j} n_{mij}\left[g_{mt}\left(\sum_{\tau=i}^{t} w_{mij\tau}\right) - g_{mt}\left(\sum_{\tau=i}^{t-1} w_{mij\tau}\right)\right] \tag{1}$$

such that:

- At most one harvest system in each time block:

$$\sum_{c\in\mathbf{C_b}}\sum_{k\in\mathbf{O}_c} x_{kb} \le 1, \qquad \forall b\in\mathbf{B} \tag{2}$$

- Minimum productivity:

$$\sum_{m\in\mathbf{M}_g}\sum_{i\in\mathbf{T}:i\le t}\sum_{j\in\mathbf{T}:j\ge t} n_{mij}\left[f_m\left(\sum_{\tau=i}^{t} w_{mij\tau}\right) - f_m\left(\sum_{\tau=i}^{t-1} w_{mij\tau}\right)\right]$$
$$\ge \sum_{k\in\mathbf{O}_c} W_{gkt}x_{kb}, \qquad \forall b\in\mathbf{B}, t\in\mathbf{T}_b, c\in\mathbf{C}_b, g\in\mathbf{G} \tag{3}$$

- There is a limit on scheduled machine hours.

$$w_{mijt} \le H, \qquad \forall m\in\mathbf{M},\ i,j,t\in\mathbf{T}: i\le t, j\ge t \tag{4}$$

- Variable signs:

$$\mathbf{x}\in\{0,1\},\ \mathbf{n}\ \mathit{integer},\ \mathbf{w}\ge 0 \tag{5}$$

Piecewise linear functions f_m and g_{mt} will be linearised so that FHER is a mixed-integer linear program. We describe the linearisation below.

2.1 Modelling functions f_m and g_{mt}

Recall that we give all machines bought and salvaged together the same workloads, thus we use the notation (m, i, j) to refer to machines of type m which are bought in time period i and salvaged in time period j. Also we assume that the linear intervals of functions f_m and $g_{mt}, \forall t\in\mathbf{T}$ are the same. We define the parameters variables below to capture the piecewise linear aspect of functions f_m and g_{mt}:

Z_m = the number of intervals for the functions for machine $m\in\mathbf{M}$
ζ_{mz} = the right endpoint of interval $z\in\{0,\dots,Z_m\}$ for machine type $m\in\mathbf{M}$, so that f_m and g_{mt} have intervals

$[\zeta_{m0}, \zeta_{m1}]$, $[\zeta_{m1}, \zeta_{m2}]$,$\ldots$, $[\zeta_{m(Z_m-1)}, \zeta_{mZ_m}]$ (let $\zeta_{m0} = 0$)

Γ_{mz} = the uptime and efficiency of machine type $m \in \mathbf{M}$ when its age is in interval $z \in \{1, \ldots, Z_m\}$

Ω_{mtz} = the age-dependent cost per scheduled hour of machine type $m \in \mathbf{M}$ when its age is in interval $z \in \{1, \ldots, Z_m\}$ (adjusted for inflation at time t)

Function f_m can be expressed as follows, in terms of the parameters described above:

$$f_m(\sigma) = \begin{cases} \Gamma_{m1}\sigma, & 0 \leq \sigma \leq \zeta_{m1}; \\ \Gamma_{mk}\left(\sigma - \sum_{l=1}^{k-1} \sigma_{ml}\right) + \sum_{l=1}^{k-1}[\Gamma_{ml}(\zeta_{ml} - \zeta_{m(l-1)})], & \zeta_{m(k-1)} \leq \sigma \leq \zeta_{mk} \\ & for\ k = 2, \ldots, Z_m; \end{cases} \tag{6}$$

Function g_{mt} can be expressed similarly by writing Ω_{mtz} in place of Γ_{mz}. We define the following decision variables to describe the piecewise linear functions:

α_{mijtz} = 1 if the machines (m, i, j) are in the interval $z \in \{1, \ldots, Z_m\}$ at time t, 0 otherwise;

y_{mijtz} = the coefficient of point $(\zeta_{mz}, f_m(\zeta_{mz}))$ or $(\zeta_{mz}, g_{mt}(\zeta_{mz}))$ at time t, for machines (m, i, j), where $z \in \{0, \ldots, Z_m\}$

We can replace terms involving f_m, g_{mt} and w_{mijt} in the objective function (1) and constraints (3), (4), (5) as follows:

$$n_{mij}\left[f_m\left(\sum_{\tau=i}^{t} w_{mij\tau}\right) - f_m\left(\sum_{\tau=i}^{t-1} w_{mij\tau}\right)\right] = \sum_{z=1}^{Z_m} y_{mijtz} f_m(\zeta_{mz}) - \sum_{z=1}^{Z_m} y_{mij(t-1)z} f_m(\zeta_{mz}) \tag{7}$$

$$n_{mij}\left[g_{mt}\left(\sum_{\tau=i}^{t} w_{mij\tau}\right) - g_{mt}\left(\sum_{\tau=i}^{t-1} w_{mij\tau}\right)\right] = \sum_{z=1}^{Z_m} y_{mijtz} g_{mt}(\zeta_{mz}) - \sum_{z=1}^{Z_m} y_{mij(t-1)z} g_{mt}(\zeta_{mz}) \tag{8}$$

$$0 \leq w_{mijt} \leq H \Rightarrow 0 \leq \sum_{z=1}^{Z_m} y_{mijtz}\zeta_{mz} - \sum_{z=1}^{Z_m} y_{mij(t-1)z}\zeta_{mz} \leq Hn_{mij} \tag{9}$$

and append the following constraints to model FHER:

- Only one interval is selected for machine (m, i, j) in time period t:

$$\sum_{z=1}^{Z_m} \alpha_{mijtz} = 1, \qquad \forall m \in \mathbf{M},\ i, j, t \in \mathbf{T} : i \leq t, j \geq t \tag{10}$$

- The coefficients sum to the number of machines (m, i, j) in time period t:

$$\sum_{z=0}^{Z_m} y_{mijtz} = n_{mij}, \qquad \forall m \in \mathbf{M},\ i, j, t \in \mathbf{T} : i \leq t, j \geq t \tag{11}$$

- The α_{mijtz} variables ensure that if interval z is selected, the $y_{mijt(z-1)}$ and y_{mijtz} are the only variables that are non-zero:

$$y_{mijt0} \leq M\alpha_{mijt1}, \qquad \forall m \in \mathbf{M},\ i,j,t \in \mathbf{T} : i \leq t, j \geq t \tag{12}$$

$$y_{mijtz} \leq M\left(\alpha_{mijtz} + \alpha_{mijt(z+1)}\right), \quad \forall z \in \{1,\ldots,Z_m - 1\}, m \in \mathbf{M},\ i,j,t \in \mathbf{T} : i \leq t, j \geq t \tag{13}$$

$$y_{mijtZ_m} \leq M\alpha_{mijtZ_m}, \qquad \forall m \in \mathbf{M},\ i,j,t \in \mathbf{T} : i \leq t, j \geq t \tag{14}$$

where M is a sufficiently large number.

2.2 An alternative MILP formulation

Both Forestry and Mining have common operational features and equipment selection criteria. (Burt 2008; Burt et al. 2010)'s work on equipment selection for the mining industry also uses MILP to optimally select equipment over multiple time periods to satisfy production requirements. Model FHER considers purchase and salvage pairs of years, whereas (Burt 2008; Burt et al. 2010)'s model uses a binary matrix to record when a machine was owned. One key difference is that (Burt 2008; Burt et al. 2010)'s work assumes that the intervals of functions f_m and g_m are longer than the number of hours in the working year. This means that at most two intervals can be active in any given year, which simplifies the model considerably. Even so, their approach was only tractable for a horizon of 4 or 5 time periods with a commercial solver.

2.3 A simplified model, FHER-S

We can, alternatively, formulate a problem where machine running costs are approximated by assuming that the machines are run at full capacity. To do this, let

P_{ma}	:	the productivity of machine $m \in \mathbf{M}$ at age a
C^M_{mij}	:	the unit cost of purchasing machine $m \in \mathbf{M}$ at the start of period $i \in \mathbf{T}$, maintaining and operating it between periods i to j (inclusive) and salvaging it at the end of period $j \in \mathbf{T}$
s_{mat}	:	the number of machines $m \in \mathbf{M}$ of age a available for use in time period $t \in \mathbf{T}$.

Here, we wish to maximise

$$\sum_{c\in\mathbf{C}} \sum_{k\in\mathbf{O_c}} R_c x_{kb} - \sum_{m\in\mathbf{M}} \sum_{i,j\in\mathbf{T}} C^M_{mij} n_{mij} \tag{15}$$

such that:

- At most one harvest system for each contract:

$$\sum_{k\in\mathbf{O}_c} y_{kb} \leq 1, \qquad \forall c \in \mathbf{C} \tag{16}$$

- Meet the production targets (for setups in use):

$$\sum_{m \in \mathbf{M}_g} \sum_{0 \leq a \leq t-1} P_{ma} s_{mat} \geq \sum_{c \in \mathbf{C}} \sum_{k \in \mathbf{O_c}} W_{gkt} y_{kb}, \quad \forall t \in \mathbf{T}, g \in \mathbf{G} \tag{17}$$

- Number of machines:

$$s_{mat} = \sum_{i \in \mathbf{T}: i \leq t, t-i=a} \sum_{j \in \mathbf{T}: j \geq t} n_{mij}, \quad \forall t \in \mathbf{T}, m \in \mathbf{M}, a \in \mathbf{N}, a \leq T \tag{18}$$

  where $\mathbf{N}$ is the set of machine ages.

Let this formulation be FHER-S. One problem with this model relates to the fact that we include the full costs of running machines even when they are not required to run at capacity.

3 A Prototype Decision Support Tool

The decision support tool is a progression from the widely known *Machine Rate*. A Machine Rate is the cost per hour of owning and operating a piece of equipment, averaged across its life and including the purchase cost minus the salvage value. (Brinker et al. 2002; Murphy and Associates 2008) illustrate such an approach. They are relatively simple, and even though the amount of data required to calculate a Machine Rate is small, it is still the case that many of the inputs are typically rules of thumb. Machine Rate calculations commonly in use can be adapted to generate input for FHER-S with little added effort, with the disclaimer that more empirical data will need to be given for more accurate modelling to take place.

The FHER-S model can be solved quickly for realistically sized problems. As such it is currently the underlying model used by the prototype tool. The FHER model remains computationally challenging, but works as a basis for heuristics to be developed in the future. At least until these heuristics are implemented, the decision support tool has a relatively simple design. It displays an interface to the user, manages the data that is input and calculates the input that the solver requires. The finished prototype will further be required to invoke the solver directly and present the generated schedule back to the interface. The tool is important because these models require a significant amount of data in order to run and managing that data well is essential to these models being usable.

One strength of the tool is that it has its own internal representation of the model that is in use, including the data. This allows it to use different solvers – each solver only requires an interfacing layer to be written rather than a rewrite of the tool. For the purposes of testing it uses the FICO Xpress-Optimizer solver, but has a layer that will allow it to use the open source solvers GLPK or SYMPHONY when it is released to the industry. The tool is written in F# (mostly) and C# (GUI) and and uses the .NET Framework. Figure 1 shows screenshots from the tool's GUI whereby the financial scenario information and the attributes of a Feller Buncher is entered.

The tool is scheduled to be evaluated by contractors by the end of year 2010. Therefore we are not able to report on numerical results or feedback from contractors at this stage.

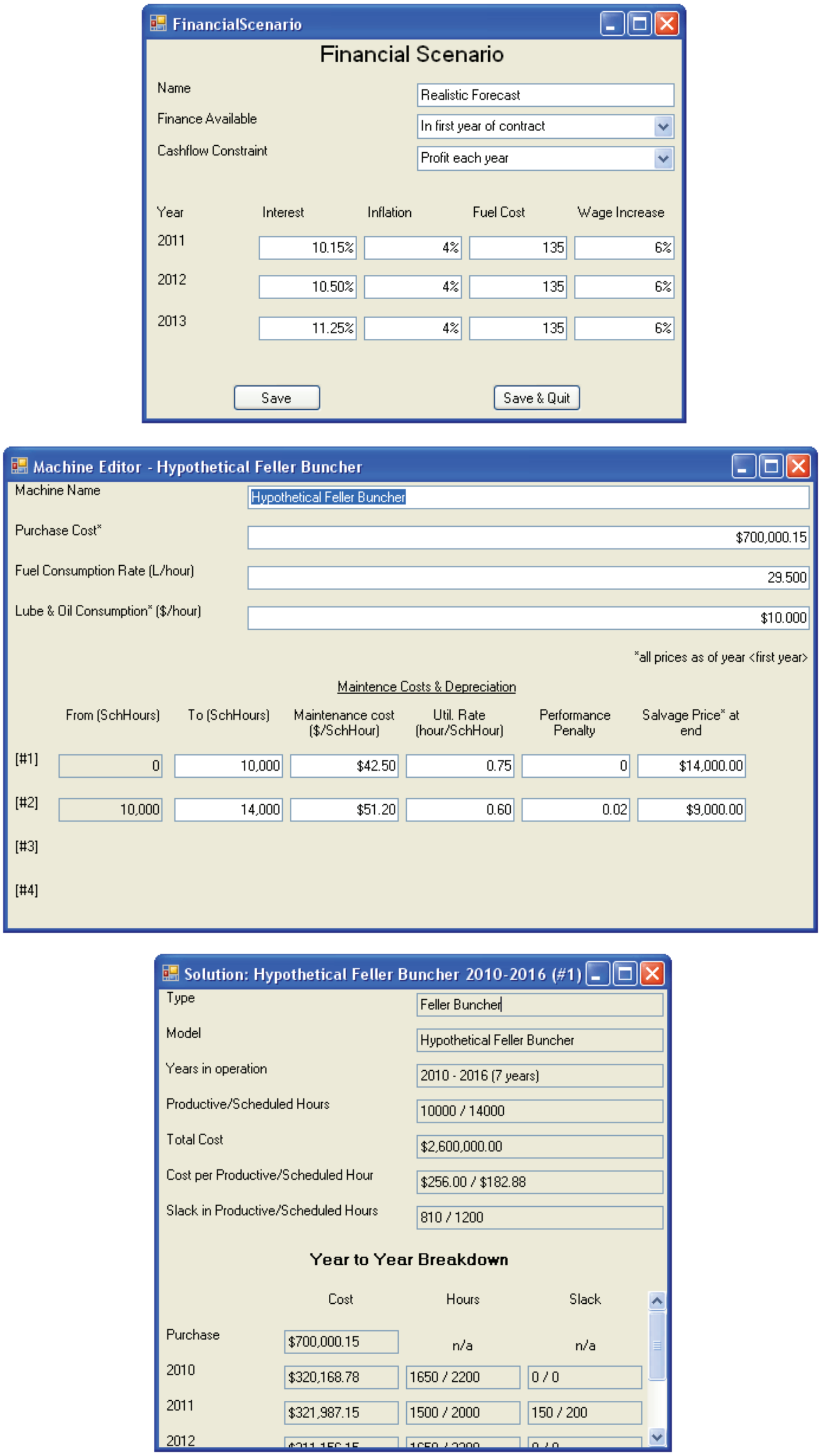

Figure 1: The Prototype Decision Support Tool.

4 Conclusions

We presented a prototype decision support tool for the equipment replacement problem in forestry, together with its underlying optimisation models. Much work remains to be done for effective solutions to this problem to be mature and to be adopted by the industry, but this paper has laid a modelling framework for further work. Particular challenges to be met include:

- Further development and refinement of the decision support tool.
- Productivity estimates and better costing parameters for harvesting machinery.
- An accurate but fast heuristic for the FHER model.
- Communication of results with the industry and researchers in forestry.

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